The Definition of Dark, Grey and Bright Time at ING

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The algorithm used to partition telescope time at ING into dark, grey and bright categories, which is based on the moonless fraction of each night, is shown to be an inconsistent measure of sky surface brightness because it ignores the effect of the daily change in longitude of the Sun on the illuminated fraction, k, of the Moon. This is manifested by frequently-occurring large differences in k on, for example, first and last bright nights within a lunation. In $\sim 15\%$ of all first/last bright nights in the *same* lunation, $\Delta k \geq 0.25$, and Δk can be as high as 0.35. This corresponds to differences in the sky surface brightness of $V_{sky} \sim 1$ magnitude/arcsec^2 *with the Moon in the sky for the same fraction of the night.*

An improved partitioning scheme, in terms of the lunar illuminated fraction acting as a proxy for the sky brightness component from scattered moonlight, and quantified in terms of its impact on observing efficiency, is derived:

$$\text{Dark: } 0.00 \leq k < 0.25; \text{ Grey: } 0.25 \leq k < 0.65; \text{ Bright: } 0.65 \leq k \leq 1.00$$

This scheme has a smaller range than the legacy method in the number of nights in each category over long baselines, and averaged over a Saros cycle yields the same number of dark nights, four more grey nights and four fewer bright nights *per semester*. The range in V_{sky} at low airmass, corresponding to high galactic- and ecliptic latitudes at solar minimum, and at an angular separation from the Moon of $\sim 90°$, is $21.2 - 21.9$ magnitudes/arcsec^2 for dark time, and $19.9 - 21.2$ and $18.0 - 19.9$ magnitudes/arcsec^2 respectively for the *moonlit parts* of grey and bright time. For the *mean* sky, i.e. over all latitudes $\sim 90°$ from the Moon, these ranges are ~ 0.5 magnitude/arcsec^2 *brighter* because of the enhanced contributions of airglow, zodiacal light and starlight.

1 Introduction

A glance at the night sky in La Palma indicates that the method used to partition telescope time into dark, grey and bright categories is problematic. The sky surface brightness depends primarily on the illuminated fraction of the Moon, but the fractional illumination on, for example, the first bright night of a lunation varies noticeably from lunation to lunation. To illustrate this, 6/7th November 2000 and 29/30th May 2001 were both designated as first bright night in a lunation, but the illuminated fraction of the Moon at 0h UT on these nights was 75% and 51% respectively. This difference in illuminated fraction corresponds to a difference in sky brightness of ~ 1 magnitude/arcsec^2.

Inconsistent partitioning of lunations can lead to a mis-match between programme *requirements* and *actual* conditions; a programme given a specific grey-time award, based on some assumed "typical" grey-time sky background, can be scheduled in significantly brighter conditions, and vice versa, and this is detrimental to observing efficiency. Also, if the number of, for example, grey nights in a semester is not adequately quantified, this impacts the supply-and-demand for such nights.

These considerations, and the need to have a good understanding of the *distribution* of sky brightness levels as the WHT moves towards a significant fraction of queue-scheduled observing, prompted a reappraisal of the legacy method used at ING, and of a better alternative to it. The purpose of this document is to derive a consistent and sensible partitioning of telescope time into dark, grey and bright categories for classical telescope scheduling. A related document will discuss the distribution of sky brightness conditions as a precursor to efficient queue loading for queue-scheduled observing, and tools to give on-the-fly prediction of sky brightness in arbitrary lines-of-sight and at arbitrary times, in order to optimise the match of sky conditions with Observing Block requirements.

2 Partitioning: The Legacy Method

A description of the algorithm used, defined and computed by the RGO up to 2001, is not available. Analysis of legacy software indicated that a quantity called *Moon phase*, d, is partitioned such that when $d \geq 0.6485$ the night is dark, when $d \leq 0.35$ the night is bright, and remaining nights are grey. Inspection of the input files to this software shows that d is the number of hours of astronomical darkness for which the Moon is at zenith distance $z > 90$, divided by the number of hours of *nautical* darkness. This parameter therefore differs from the fraction of the night that is dark by a scaling factor of ~ 0.9, so that $0 \leq d \lesssim 0.9$.

2.1 Moonless Fraction of the Night

Although there is merit to using the fraction of each night that is moonless to define dark time (it constrains the duration of the moonlit part of every night in dark time), it *does* permit a variation in sky brightness in this $\lesssim 30\%$ moonlit part, and there is less merit in this approach as the Moon ages, i.e. in defining grey and bright time. As the sky brightens the focus should change to quantifying the effects of the higher background on observing efficiency. This algorithm can therefore be criticized in several aspects:

- it does not relate *directly* to the illuminated fraction of the Moon, i.e. to the primary *cause* of the enhancement of the sky background at some point over moonless conditions.
- it is internally inconsistent: a given bright period can be asymmetrical with respect to full Moon by up to ~ 3 nights, and therefore up to ~ 3 grey nights in one lunation can be *brighter* than the first (or last) bright night in the *same* lunation.
- the baseline value of the metric itself, i.e. the ratio of astronomical and nautical darkness, varies from $d_0 \sim 0.86$ in Summer to $d_0 \sim 0.92$ in Winter.
- it yields, on average, considerably fewer grey than dark and bright nights; the number of grey nights could increase at the expense of bright nights, thereby smoothing pressure factors.

Table 1 lists two widely-spaced examples which illustrate the inconsistencies in the V sky brightness, computed at representative points $\sim 90^\circ$ through the zenith from the Moon, which can result from this partitioning scheme. It is seen that grey periods immediately preceding and succeeding a bright period can differ very significantly in sky brightness. For example, the two grey nights bounding a bright period can differ in sky brightness by up to ~ 1.3 magnitude/arcsec^2 *and* have the Moon present for the *same* fraction of the night (by definition of this partitioning algorithm). Also, grey nights can be up to ~ 1 magnitude/arcsec^2 *brighter* than first/last *bright* nights in the adjacent bright period, and have the Moon present for a *similar* fraction of the night. This has a significant impact on observing efficiency; for example, observing (in the visible) a $V \sim 21$ magnitude point source in a background enhanced from $V \sim 20.6$ to $V \sim 19.6$ magnitudes/arcsec^2, corresponds to a *loss* in observing efficiency of $\sim 40\%$ (Appendix 2).

Table 1

Night	k	V_{sky}	Category
28/07/1990	0.45	20.5	Grey
29/07/1990	0.54	20.2	Grey
30/07/1990	0.63	19.9	Grey
31/07/1990	0.72	19.6	Grey
01/08/1990	0.80	19.4	First Bright
12/08/1990	0.58	20.1	Bright
13/08/1990	0.46	20.5	Last Bright
14/08/1990	0.35	20.9	Grey
15/08/1990	0.24	21.2	Grey
16/08/1990	0.15	21.5	Grey
27/04/2001	0.20	21.4	Grey
28/04/2001	0.31	21.0	Grey
29/04/2001	0.42	20.6	Grey
30/04/2001	0.53	20.3	First Bright
01/05/2001	0.65	19.9	Bright
11/05/2001	0.81	19.3	Last Bright
12/05/2001	0.72	19.6	Grey
13/05/2001	0.64	19.9	Grey
14/05/2001	0.54	20.2	Grey
15/05/2001	0.45	20.5	Grey

These inconsistencies occur frequently. For example, over a long time baseline the difference in fractional lunar illumination, Δk, between first and last bright nights of the *same* lunation can be as high as 0.35, and $\Delta k \geq 0.20$ in $\sim 30\%$ of lunations, $\Delta k \geq 0.25$ in $\sim 15\%$ of lunations and $\Delta k \geq 0.30$ in $\sim 5\%$ of lunations. In $\sim 43\%$ of lunations $\Delta k \leq 0.10$. It is emphasised that for each of these asymmetric first/last bright nights, the Moon would be in the sky for the same fraction of the night *by definition of the partitioning algorithm.*

2.2 Illuminated Fraction of the Moon

The illuminated fraction, k, of the Moon's disc presented to the Earth is dependent on the relative positions of the Moon *and* Sun, and is given by

$$2k = 1 + \cos i \tag{1}$$

where i is the lunar phase angle, i.e. the angular separation of the Earth and Sun as viewed from the Moon, and $0 \leq i \leq 180$. It is approximated to within ~ 0.001 (Meeus, 1998) by substituting

$$\cos i \sim -\cos\beta\cos(\lambda - \lambda_{\odot}) \tag{2}$$

where β is the Moon's latitude, and λ, $\lambda_{\odot}$ are the longitude of the Moon and Sun respectively. The local hour angle, H_0, corresponding to rising and setting of the Moon, is $\cos H_0 = -\tan\phi\tan\delta$, where ϕ is the observer's latitude, and δ is the Moon's declination, and the Moon is below the horizon for a period of $D_m = 24(\pi - H_0)/\pi$ hours. Partitioning the fraction of D_m which occurs in astronomical darkness therefore correlates strongly with partitioning the Moon's longitude, λ (the Moon's latitude $|\beta| \lesssim 5.3°$), *but ignores the change in longitude of the Sun by* $\sim 1°$ *per day.*

Suppose the Moon's longitude is partitioned at 90° (M_1) and at 270° (M_2) beyond new Moon (Figure 1). When the Moon is at M_1 the longitude of the Sun, now at S_1, has also increased, and $(\lambda - \lambda_\odot)_{M_1} < 90°$, so that by (1) and (2) *less* than 50% of the Moon's disc is illuminated. When the Moon is at M_2, the Sun is at S_2, and $(\lambda - \lambda_\odot)_{M_2} > 90°$, and so *more* than 50% of the Moon's disc is illuminated. This is the basic mechanism by which the envelopes of dark, grey and bright categories, as defined by the fraction of the night that is dark, become skewed in terms of sky surface brightness.

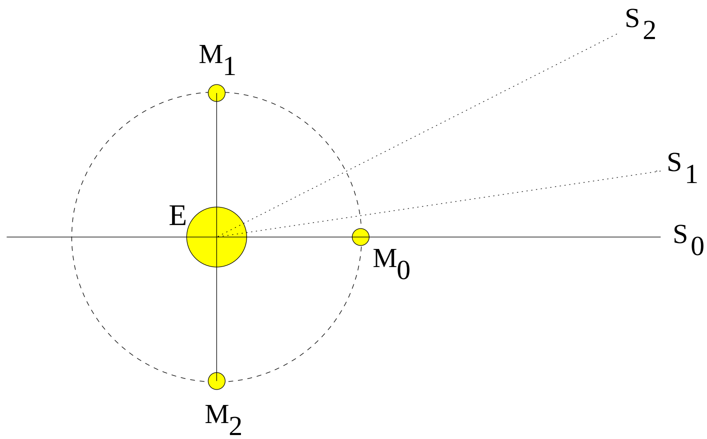

Figure 1. Relative longitudes of the Moon and Sun in a lunation

However, the real situation is more complex; the Moon's orbit has a small eccentricity of ~ 0.055, and many perturbations act upon it. The major effects of this in the present context are that the daily rate of the Moon's longitude is not constant, the line of apsides joining perigee and apogee rotates with a period of ~ 8.6 years in a prograde sense, and the line of nodes rotates with a period of ~ 18.6 years in a retrograde sense. The effect of ignoring the daily motion of the Sun in longitude is therefore *modulated* by these non-linear aspects of the Moon's motion. It is this that causes the observed *variation* in consistency of the definition of dark, grey and bright time over several seasons, so that the first bright night in a lunation can be similar to, or either much *fainter* or *brighter* than, the last bright night in the same lunation. For example, if the Moon reaches $\sim M_1$ *late* and reaches $\sim M_2$ *early* ($\sim M_1$ and $\sim M_2$ refer to *actual* partitioning longitudes, not necessarily at $\lambda = \pm 90°$ from new Moon), then $(\lambda - \lambda_\odot)_{M_1} > (\lambda - \lambda_\odot)_{M_2}$, and first bright night will be *brighter* than last bright night, and vice versa. These arguments of course also apply to the partition between dark and grey categories.

Partitioning classically-scheduled time in terms of the number of days from new Moon is also inconsistent in terms of lunar phase and sky brightness; the above-mentioned aspects of the lunar orbit conspire so that times of first and second quadratures can vary by almost 2 days, i.e. from ~ 6 days to ~ 8 days from new Moon. Therefore "days from new Moon" does not sample a consistent illuminated fraction, although the inconsistencies are less marked than those described in Section 2.1.

The direct way to partition classically-scheduled time is in terms of sky brightness itself, since it is this which impacts the signal-to-noise ratios for observations of a given integration time.

3 Sky Surface Brightness Estimation

The main contributors to the moonless sky optical surface brightness in a line-of-sight and in a specific bandpass are airglow, the zodiacal light and starlight. Benn and Ellison (1998) (see Appendix 1) find the high galactic latitude, high ecliptic latitude, zenith V sky brightness at solar minimum to be $V = 21.9$ magnitudes/arcsec^2. The sky is brighter at low latitudes by ~ 0.4 magnitude/arcsec^2, and at higher airmasses by ~ 0.3 magnitude/arcsec^2 (at $X \sim 1.5$), and due to variable solar activity, the airglow component is brighter by ~ 0.4 magnitude/arcsec^2 at solar maximum. There is no dependence on extinction, A_V, for $A_V < 0.25$ magnitude/airmass.

In the moonlit sky, the main contributions to the optical sky brightness are the illuminated fraction of the Moon's disc, followed in importance by the angular distance between the Moon and the line-of-sight. Smaller dependences are on the Moon's airmass and atmospheric extinction. The contribution from scattered moonlight when the illuminated fraction of the Moon is $\gtrsim 0.15$ exceeds the spatial variations from airglow, zodiacal light and starlight, and so to partition classically-scheduled telescope time it is sufficient to partition by the illuminated fraction of the Moon's disc, acting as a proxy for sky surface brightness due to scattered moonlight.

3.1 Scattered Moonlight

Two scattering mechanisms dominate the background from moonlight; Mie scattering by aerosols and Rayleigh scattering by molecules. Mie scattering is highly forward, and so Rayleigh scattering dominates for scattering angles (i.e. angular distance from the Moon) $\gtrsim 90°$.

Krisciunas & Schaefer (1991) (see Appendix 1) give scattering formulae to compute the contribution of moonlight to the sky background at some airmass as a function of lunar phase, lunar zenith distance, distance from the Moon and extinction. These are normalised to JKT *measurements* of the moonlit sky made by Chris Benn in 1998, by multiplying by a scaling factor of $\times 2.4$.

The presence of dust will also enhance the sky background, both in the moonless sky, and more significantly in the present context, in the moonlit sky; this is not considered in these scattering formulae.

The mean ΔV_{moon} is computed from these formulae with the Moon at zenith distance[1] 60°, as a function of lunar illuminated fraction in increments of 0.1, and at scattering angles ranging from 30° to 120° in increments of 5°, measured both in the azimuthal direction and in altitude (Figure 2). The computed differences in both directions at a given scattering angle are only a few per cent.

The effect of decreasing the Moon's zenith distance on ΔV_{moon} at a given distance from it is small, $\lesssim 0.1$ magnitude, i.e roughly same size as the points in Figure 2 (ΔV_{moon} *does* however

[1]Zenith distance of 60° for the Moon is chosen in these calculations so that scattering angles of up to 120° are accommodated at airmasses ≤ 2.

fall off rapidly by ~ 1 magnitude/arcsec^2 when the Moon is low on the horizon).

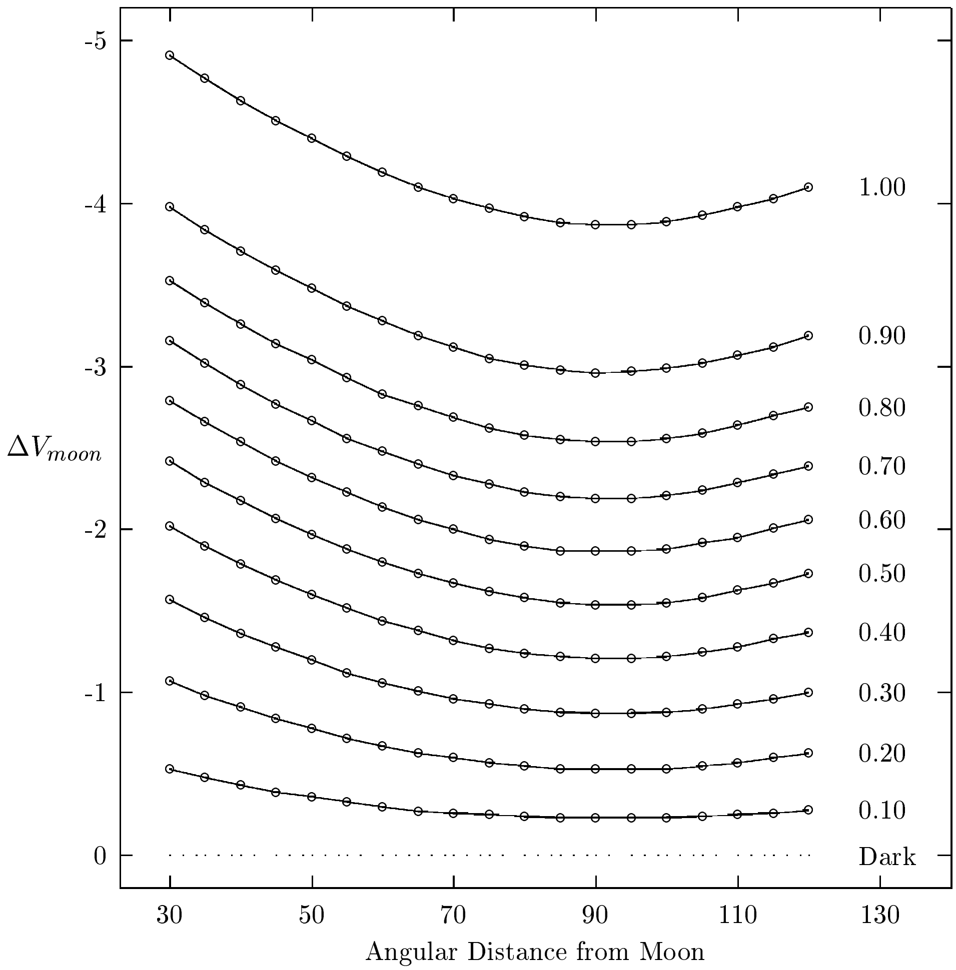

Figure 2: The brightening of the dark sky ($V = 21.9$ magnitudes/arcsec^2, corresponding to low airmass, high ecliptic and galactic latitude, and solar minimum) by scattered moonlight, calculated as a function of lunar phase and angular distance from the Moon.

Therefore, Figure 2 forms a consistent basis for quantifying the effects of moonlight on the sky background at specific scattering angles from the Moon. Several aspects of this are noteworthy:

- As the scattering angle increases, the contribution of scattered moonlight to the sky background decreases up to $\sim 90°$, and then begins to increase again when Rayleigh scattering dominates. Therefore, in the presence of moonlight, a good strategy is to observe in a broad annulus centred $\sim 90°$ from the Moon[1] whenever possible, in order to minimise the effects of scattered moonlight.
- The gradient of scattered moonlight is remarkably flat at scattering angles $\sim 90°$. In fact, even at full Moon, the range in scattered moonlight within $70° - 110°$ of the Moon is $\Delta V_{moon} \lesssim \pm 0.1$. Therefore, to quantify the contribution from scattered moonlight to

[1]Scattered moonlight $\sim 90°$ from the Moon is highly polarised, and Baldry & Bland-Hawthorn (2001) proposed the use of a polariser to allow 'dark-time' observations to be made in bright conditions in this region of the sky.

sky brightness partitioned by illuminated lunar fraction, it is sensible to do so in terms of ΔV_{moon} computed $\sim 90^\circ$ from the Moon.

- The illuminated lunar fraction assumes greater importance in determining the sky background than distance from the Moon, for Moon separations $\gtrsim 50^\circ$. The change in lunar illuminated fraction is ~ 0.1 per day in the neighbourhood of quadratures, and therefore this emphasises the importance of deriving consistent partitioning between grey and bright categories; inconsistent partitioning is in general not compensated for by the distribution of targets on the sky in relation to the Moon.
- The sky brightness approaching full Moon, i.e. zero lunar phase angle, increases strongly due to the opposition effect, which arises from a combination of shadow-hiding (the shadows of lunar particles are occulted by the particles themselves) and coherent backscattering (multiple scattering of sunlight off lunar dust grains, predominantly in the backward direction to the incident sunlight).

For a given lunar phase, the fraction of the night for which the Moon is in the sky can vary by as much as $\sim 25\%$, for the same reasons that the fraction of the night which is moonless does not consistently estimate lunar phase (Section 2.2). For example, at $k = 0.65$ the Moon can be in the sky for $\sim 65\%$ to $\sim 90\%$ of astronomical darkness. This could be taken into account for each night by weighting the moonlit sky background by this fraction to give a "mean" background for the night, but on balance it is considered better to partition solely in terms of the sky brightness with the Moon in the sky, i.e. to partition in terms of the *worst case* sky brightness $\sim 90^\circ$ from the Moon.

3.2 Relative Observing Efficiency

The impact of an enhanced sky background on an observation of a given target is to reduce *observing efficiency*; in the elevated background a longer integration is needed to achieve the same signal-to-noise ratio (see Appendix 2). Scheduling observations in darker conditions than assumed, is, of course, also inefficient; awards are made, and should be scheduled, in the context of a commonly understood range of background levels.

In Figure 3, the observing efficiency relative to the V=21.9 moonless sky, $\varepsilon_{m,n}$, i.e. the ratio of integration times which gives the same signal-to-noise ratio, is plotted as a function of lunar phase (as a proxy for sky brightness) for background-limited observations, and for a point-source with $V = 21$ in median seeing ($0.7''$) in an optimal aperture. The curve for $V = 21$ is essentially linear over most of the lunar cycle. The sharp drop in relative efficiency at $k \sim 0.95$ is due to the abrupt increase in sky brightness caused by the opposition effect.

This figure demonstrates the consequence of inconsistent partitioning in terms of observing efficiency. For example, for the 1990 example in Table 1, the grey night immediately preceding the bright period has a lunar phase, $k = 0.72$, and the grey night immediately succeeding the bright period has lunar phase $k = 0.35$. For background-limited and $V = 21$ point-source observations on the former night, the observing efficiency, excluding duty cycle considerations, is less than half what it would be on the latter darker night, for the $\sim 70\%$ of each night for which the Moon is in the sky. Similarly, the same observations on at least two of the preceding grey nights are $\gtrsim 35\%$ *less* efficient than on at least two of the *bright* nights in this period. The legacy method is therefore internally inconsistent in how it apportions the three categories.

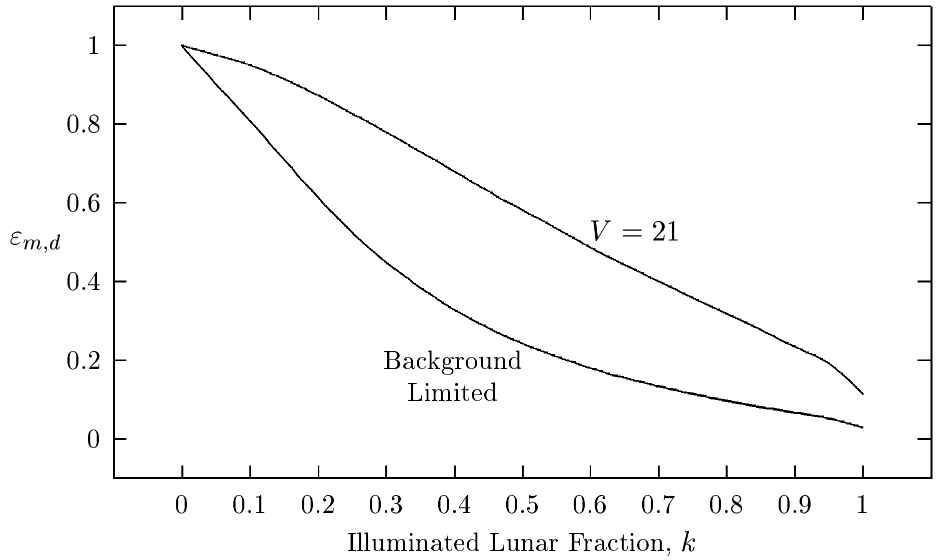

Figure 3: Relative observing efficiency, $\varepsilon_{m,n}$, computed 90° from the Moon as a function of illuminated lunar phase for the background-limited case, and for a point source with $V = 21$ in median seeing of $0.7''$, and in an optimal aperture.

3.3 Partitioning By Sky Surface Brightness

The definition of grey and bright thresholds is to some extent arbitrary. What is important is that the definition is both *sensible* and *consistent*, and that it is understood and agreed in terms of its impact on observing efficiency by applicants and TAC's, and adhered to in the scheduling process.

One of the goals of this redefinition, consistency in terms of sky brightness *and* the range in the numbers of nights in each category over long baselines, is met by partitioning on sky brightness (illuminated lunar fraction). In terms of sensibleness, the numbers of nights in each category should not be greatly different from the legacy method, but a small increase in the number of *grey* nights per semester, at the expense of bright nights, is desirable to better match demand.

Averaged over a Saros cycle ($\sim$ 37 semesters), the *same* number of dark nights, four *additional* grey nights and four *fewer* bright night *per semester* result from the adopted partitioning scheme:

$$\begin{aligned} Dark: &\quad 0.00 \le k < 0.25 \\ Grey: &\quad 0.25 \le k < 0.65 \\ Bright: &\quad 0.65 \le k \le 1.00 \end{aligned}$$

where the illuminated fraction is computed for 0h UT. Illuminated fraction changes by ~ 0.1 per day in the region of these thresholds, and this "resolution effect" means that inconsistencies in illuminated fractions are constrained to be $\lesssim 0.1$ at the dark, grey and bright boundaries. The legacy method over long baselines permits $0.12 \lesssim k \lesssim 0.76$ for grey time, but still has four *fewer* grey nights on average than the sky brightness scheme.

In Table 2, the range and mean per lunation in the number of nights in each category over a Saros cycle is compared for the legacy method [old] and the sky surface brightness method [new]. This table also gives the predicted *zenith* V_{sky} magnitude range for dark time and for the moonlit parts of the grey and bright categories as defined by the sky surface brightness method, *corresponding to high ecliptic and galactic latitude, and solar minimum.*

Table 2

Category	Nights [old]	Nights [new]	V_{sky}
Dark	8-12 $\langle 9.6 \rangle$	8-11 $\langle 9.6 \rangle$	21.2-21.9
Grey	5-9 $\langle 7.2 \rangle$	6-9 $\langle 7.9 \rangle$	19.9-21.2
Bright	11-15 $\langle 12.7 \rangle$	10-14 $\langle 12.0 \rangle$	18.0-19.9

In *arbitrary* lines-of-sight these ranges will be brighter by ~ 0.5 magnitude because of the spatial variation of the moonless sky, including the airmass dependence of airglow, i.e. the *mean* moonless sky background is brighter by ~ 0.5 magnitude than the figure of 21.9 magnitudes/arcsec^2 appropriate to high latitudes and small airmasses at solar minimum.

As Benn and Ellison (1998) note, the effect of the enhanced airglow at *solar maximum* is to increase the *mean* sky brightness by ~ 0.4 magnitude, which is approximately of the same size as the nightly change in scattered moonlight. Therefore, near solar maximum it may be wise to be somewhat more conservative in partitioning k, for example $k < 0.2$ for dark time, and $k < 0.6$ for grey time, but given the overall uncertainties involved, this is probably not essential.

In terms of observing efficiency relative to the dark sky, these thresholds correspond to a relative observing efficiency of ~ 0.85 at the threshold of grey time, and ~ 0.45 at the threshold of bright time for $V = 21$ point sources, and ~ 0.53 and 0.15 respectively for background-limited observations.

4 Summary

The algorithm used to define dark, grey and bright categories of telescope time at ING does not relate directly to the fractional illumination of the Moon, and is inconsistent in its representation of the sky surface brightness. It is shown that this occurs because it is does not take into account the daily change in longitude of the Sun, and consequently does not map consistently onto the lunar illuminated fraction.

A method for better partitioning sky brightness, quantified in terms of the impact on observing efficiency, is derived using the illuminated lunar fraction, k, as a proxy for scattered moonlight: $k < 0.25$ is dark, $0.25 \leq k < 0.65$ is grey and $k \geq 0.65$ is bright. This scheme gives the same number of dark nights, four more grey nights and four fewer bright nights per semester on the average than the legacy method, and has a smaller range in the numbers in each category per lunation. The range in the V sky surface brightness at low airmass and high galactic and ecliptic latitudes, and solar minimum, in the three categories is $21.2 - 21.9$ for dark, $19.9 - 21.2$ for grey and $18.0 - 19.9$ for bright, and ~ 0.5 magnitude/arcsec^2 *brighter* than this in the *mean* sky due to the enhanced contributions of airglow, zodiacal light and starlight.

Acknowledgements

It is a pleasure to acknowledge useful discussions with Thomas Augusteijn and Rob Jeffries, and Steve Bell (NAO) for providing the lunar ephemerides on which preliminary aspects of this study are based.

References

Baldry, I.K. & Bland-Hawthorn, J., Mon. Not. R. Astron. Soc. **322**, 201-205 (2001)
Benn, C.R. & Ellison, S.L., ING Tech. Note **115**, (1998)
Krisciunas, K. & Schaefer, B.E., P.A.S.P. **103**, 1033-1039 (1991)
Meeus, J., Astronomical Algorithms, pub. Willmann-Bell (1998)

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28th February 2002
[ING Technical Note 127]

Appendix 1 Sky Background Estimation

For convenience, the formulae used to compute sky brightness in the moonless and in the moonlit sky are summarised in this Appendix.

A1.1 Moonless Sky

Benn and Ellison (1998) find the median sky brightness of the moonless sky at low airmass, high galactic latitude and high ecliptic latitude, at Sunspot minimum, to be 21.9 magnitudes/arcsec^2 in V. They parameterise the moonless sky brightness in V in terms of contributions from airglow, zodiacal light and starlight for a given line of sight as

$$V_{sky} = 27.78 - 2.5 \log(S_{air} + S_{zod} + S_{star}) \tag{3}$$

where

$$\begin{aligned}
S_{air} &= (145 + 130(S_{\odot} - 0.8)/1.2)X & \\
S_{zod} &= 140 - 90 \sin\beta & \beta < 60^\circ \\
&= 60 & \beta \geq 60^\circ \\
S_{star} &= 100 e^{-|b|/10^\circ} &
\end{aligned}$$

and X is airmass, β is ecliptic latitude, b is galactic latitude, and $S_{\odot}$ is the solar $10.7 - cm$ radio flux density in MJy. $S_{\odot}$ is a proxy for the effect of solar activity on airglow, and varies sinusoidally during the solar cycle from ~ 0.8 MJy (minimum) to ~ 2.0 MJy (maximum).

The moonless sky background is brighter at low ecliptic latitude by ~ 0.4 magnitude, brightens with zenith distance, due to the airmass dependence of airglow, by ~ 0.3 magnitude at $X \sim 1.5$, and is brighter at solar maximum by ~ 0.4 magnitude. The variation in the mean sky brightness within astronomical darkness is $\lesssim 0.1$ magnitude, and the variation with extinction, A_V, is negligible for $A_V < 0.25$ magnitude/airmass.

A1.2 Moonlit Sky

Krisciunas & Schaefer (1991) give a model for predicting the contribution of scattered moonlight to the sky brightness at some airmass, X, in terms of the Moon's illuminated fraction, its zenith distance, angular distance from the Moon (the scattering angle), and extinction. Their single-scattering model comprises two types of atmospheric scattering (i) Mie scattering by aerosols and (ii) Rayleigh scattering by atmospheric gases. Mie scattering is strongly forward, and so Rayleigh scattering dominates the process for scattering angles $\gtrsim 90^\circ$.

The change in the V-band sky brightness due to scattered moonlight is

$$\Delta V_{moon} = -2.5 \log \left[\frac{B_{moon} + B_0(z)}{B_0(z)} \right] \tag{4}$$

where $B_0(z)$ is the moonless sky brightness at a zenith distance z, and B_{moon} is the scattered moonlight in linear units, and is given by

$$B_{Mmoon} = f(\rho)\, I_0\, 10^{-0.4\, k_V\, X(z_m)} \left[1 \,-\, 10^{-0.4\, k_V\, X(z)}\right] \tag{5}$$

where $f(\rho)$ is the scattering function, I_0 is the illuminance of the Moon outside the atmosphere, k_V is atmospheric extinction, $X(z) = (1 - 0.96\sin^2(z))^{-0.5}$ is the scattering airmass of the line-of-sight and $X(z_m)$ is the airmass of the Moon. The scattering function, $f(\rho)$, where ρ is the scattering angle, is

$$f(\rho) = 10^{5.36}\left[1.06 + \cos^2(\rho)\right] + 10^{(6.15-\rho/40)} \quad (6)$$

The illuminance of the Moon at the top of the atmosphere, I_0, expressed in terms of the lunar phase angle, i, is

$$I_0 = 10^{-0.4\left[3.84 + 0.026i + 4\times10^{-9}\, i^4\right]} \quad (7)$$

Krisciunas & Schaefer estimate the accuracy of these formulae in predicting scattered moonlight in a line-of-sight to be within ~ 0.25 magnitude/arcsec^2, from observational scatter of actual *measurements* about the predictions. However, local prevailing conditions, e.g. levels of atmospheric dust, will modulate the contribution of scattered moonlight, and so on any given night the accuracy is probably closer to ~ 0.5 magnitude/arcsec^2.

For La Palma, these predictions are normalised, $\times 2.4$, to agree with *measurements* of sky brightness made at the JKT on a dust-free night in 1998, by Chris Benn. For other bands, e.g. UBRI, the sky background is estimated from the colours of moonlight, also measured at the JKT by Chris Benn. These measurements show that moonlight is approximately grey in U, B and V, but reddens at R and I, and with increasing lunar illumination:

Table 4

k	0.15	0.50	0.85	1.00
$V-R$	0.2	0.7	0.7	0.8
$V-I$	0.3	0.9	0.9	1.0

These formulae enable on-the-fly prediction of the sky brightness in arbitrary lines-of-sight and at arbitrary times, so that in queue scheduled mode a reasonable match of sky brightness *conditions* with requirements can be attempted, and exposure times optimised. Early in the night *measurement* of the sky brightness (during astronomical darkness) would improve the predictive power of these formulae for that night, as would inclusion of a simple dust-scattering model for Summer periods, when dust can be a persistent contributor to the sky background, especially in the moonlit sky.

In the context of queue observing, it is highly desirable that the pipeline processing of imaging data measures sky brightness in all images, so that its distribution and uncertainty can be better quantified, and prediction therefore made more accurate. This, together with well-calibrated instruments, is essential for efficient loading and execution of the queue.

Appendix 2 Sky Background and Observing Efficiency

The signal-to-noise ratio for an observation with a given telescope, instrument and detector is

$$s/n = \frac{QN_t t}{\sqrt{QN_t t + n_p(Qp^2 N_b t + \sigma_r^2)}} \tag{8}$$

where Q is the quantum efficiency of the detector, N_t is the *total* number of photons/second from the target incident on the detector in the seeing disc, N_b is the photons/second/arcsec^2 from the background incident on the detector, assumed uniform over the seeing disc, n_p is the number of pixels in the seeing disc, p is the pixel dimension in arcsec, t is integration time, and σ_r is the rms detector readout noise per pixel. For an imaging observation, where the instrumental/seeing profile is θ arcsec FWHM, the number of pixels in an optimal aperture of radius $\alpha = 2\theta/3$ is $n_p = \pi(\alpha/p)^2$. Assuming Poisson-dominated noise (i.e. a negligible contribution from readout noise), the signal-to-noise ratio is then

$$s/n = N_t\sqrt{\frac{Qt}{N_t + \pi\alpha^2 N_b}} \tag{9}$$

The *relative* observing efficiency, $\varepsilon_{m,d}$, of a given target due to an enhanced background from scattered moonlight, is the ratio of the exposure times, t_d, t_m, which achieves the *same* arbitrary signal-to-noise ratio (ignoring readout and duty cycle considerations). Writing the background, N_b, as the sum of the moonless sky, N_d, and the contribution from scattered moonlight, N_m, i.e. $N_b = N_d + N_m$, the relative observing efficiency is, from (10)

$$\varepsilon_{m,d} = \frac{t_d}{t_m} = \frac{N_t + \pi\alpha_d^2 N_d}{N_t + \pi\alpha_m^2(N_d + N_m)} \tag{10}$$

where α_d and α_m reflect the seeing in the dark and moonlit sky. In median seeing of $\theta = 0.7''$, for example, $\pi\alpha_d^2 = \pi\alpha_m^2 \sim 0.68$. For a target which is already background-limited in the moonless sky, $\pi\alpha_d^2 N_d \gg N_t$ and the background-limited relative observing efficiency, in *constant* seeing, is

$$\varepsilon_{m,d} = \frac{N_d}{N_d + N_m} \tag{11}$$

Therefore, with Poisson-dominated noise, relative observing efficiency of a given source in different sky backgrounds is the ratio of $(s/n)^2/t$ in each background, and for *equal* exposure times, it is the ratio of $(s/n)^2$ *achieved* in each background.

The number of photons/second from the target, N_t, incident on the detector relates to the target magnitude, m_t, for a particular telescope and instrument as

$$N_t = \Phi \times \delta\lambda \times 10^{-0.4 m_t} \tag{12}$$

where $\delta\lambda$ is the effective filter bandpass and $\Phi \sim N_0 T_a A_m T_{tel} T_{inst}$, and where N_0 is the number of photons/second/Å/cm^2 corresponding to zero-magnitude at the top of the atmosphere, T_a is the transmission of the atmosphere, A_m is the collecting area of the telescope in cm^2, T_{tel} is the throughput of the telescope and T_{inst} is the throughput of the instrument. The number of photons/second/arcsec^2 from the background, N_b, relates similarly to the sky background expressed as a magnitude, m_b magnitudes/arcsec^2.

Therefore, relative observing efficiency of a given target at some wavelength in differing levels of sky background is independent of both Φ and $\delta\lambda$, i.e of telescope and of whether the observations are imaging or spectroscopic (approximating a spectroscopic calculation by a continuum imaging calculation in a narrow bandpass and circular aperture), and it is therefore a useful parameter for quantifying the effects of partitions of telescope time as dark, grey and bright categories, and for selection of Observing Blocks in queue-scheduled mode.

To compute the relative observing efficiency it is useful to write it in terms of magnitudes. For example, for the background-limited case in V, assuming $V_{sky} = 21.9$ for the dark sky, $\varepsilon_{m,d}$ in constant seeing is

$$\varepsilon_{m,d} = 1.74 \times 10^{0.4V_{sky}-9} \tag{13}$$

Similarly, (11) can be recast in terms of V_t and V_{sky} for the non background-limited case in median seeing, as

$$\varepsilon_{m,d} = \frac{1 + 1.18 \times 10^{0.4V_t-9}}{1 + 0.68 \times 10^{0.4(V_t-V_{sky})}} \tag{14}$$

Signal-to-noise is often computed in a non-optimal aperture of diameter equal to twice the seeing. This will affect the computed values of $\varepsilon_{m,d}$ in the non background-limited case; specifically, the numerical constants in (15) change as $1.18 \rightarrow 2.68$ and $0.68 \rightarrow 1.54$, and so $\varepsilon_{m,d}$ is of course lower when computed in a non-optimal aperture.