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Accurate Calibration of Field Distortion of the JKT

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The Jacobus Kapteyn Telescope, when used at the f/8 Secondary focus, has a field of view of about 1.4 degrees and angular scale of 25.60 arcsec/mm. The Harmer-Wynne corrector makes it potentially suitable for wide-field astrometry. Even with this corrector the field has a radial distortion, which has been calculated theoretically. In order to check this theoretical effect, and to assess the quality of the astrometry once this effect is removed, a comparison has been made with a plate taken on the astrometric 26-inch telescope at Herstmonceux.

Two plates of the Crab region were compared, one taken with the 26-inch refractor at Herstmonceux in December 1986 and the other with the 1.0m JKT at the Observatorio del Roque de los Muchachos in January 1987. This plate pair was chosen as the identical field had been taken at very similar epochs, so proper motion would not affect the results. Approximately 50 stars of similar magnitude and with an even distribution across the field were selected. The measures were made with the Zeiss two-coordinate measuring machine as this provides firm plate support and has excellent optics. The field centre as given by the observing notes was marked on each plate using an overlay from CHART, and its x-y coordinates were measured to about 1 mm on the Zeiss. For the JKT the field centre differs significantly from the geometric plate centre as the plate is rectangular. It is mounted eccentrically on the telescope to leave space at one edge for calibration and identification data.

In the first comparison of the plates the 1.0m measures were not corrected for the radial distortion. Before a comparison can be made it is necessary to allow for the different scales of the two plates, the different centring on the measuring machine and for a rotation.

An estimated value of the scale differences of the two plates and the difference of the measured field centres were used as a first approximation in a least-squares fitting program to determine the x-y shift, scale difference and rotation between the plates. The differences of the transformed 26-inch measures from the uncorrected 1.0m measures are shown in Fig. 1, where the line plotted for each star is proportional to the magnitude of the difference and in the direction of the displacement. The dots mark the measured positions of the stars on the 1.0m plate, and the lines give the displacements to the transformed 26-inch positions, magnified by a factor of 100.

From the radial distortion value quoted in the optical design (Harmer, and Wynne 1976) C.M. Lowne has deduced the following for the radial distortion :

$$\Delta \vec{r} = 2.05 \text{ x } 10^{-7} r^3 \hat{r} \text{ mm}$$

and so the correction for the distortion is :

$$\Delta \vec{r} = -2.05 \text{ x } 10^{-7} r^3 \hat{r} \text{ mm}$$

where r is the distance from the optical axis in mm, and $\hat{r}$ is a unit vector in the radial direction. This expression can be expressed in rectangular coordinates as

$$\Delta x = 2.05 \text{ x } 10^{-7} r^2 (x - x_c) \text{ mm}$$

$$\Delta y = 2.05 \text{ x } 10^{-7} r^2 (y - y_c) \text{ mm}$$

where (x_c , y_c) are the coordinates of the plate centre and

$$r^2 = (x - x_c)^2 + (y - y_c)^2$$

All the coordinates must be in mm.

It is apparent that the displacements plotted in Fig. 1 do not resemble a radial displacement as given by the formula, which would give displacements that all pointed towards the optical axis (marked with a cross in Fig. 1), and would have magnitudes that increased with the distance from the optical axis. The reason is that, in fitting the 26-inch measures to the 1.0m, a scale difference has been solved for, and this has absorbed a large part of the cubic variation. To demonstrate this we have plotted in Fig. 3 the theoretical cubic radial distortion as it would appear along any line passing through the optical axis - the plot is actually for the x-axis. In fitting a scale parameter to data that contained this distortion the scale parameter would absorb much of the cubic variation, as indicated by the dashed straight line. The residuals obtained would be the difference between the cubic and the straight line, and the form of this difference is plotted in Fig. 4. This represents the actual residuals quite well. The circle marked in Fig. 1 is approximately where the residuals are zero, and corresponds to the points where the straight line intersects the cubic in Fig. 3.

For the second comparison of the plates the theoretical correction was applied to the 1.0m measures before fitting the 26-inch measures to them, The differences are plotted in Fig. 2. It can be seen that a very good fit is obtained, with an rms residual of 0.18 arcsec. This gives confidence that the theoretical formula is accurate. As a further check on the formula we have determined the magnitude of the cubic radial term by least-squares solution for the plate comparison. From the above discussion it is clear that differences plotted in Fig. 1 will be of the form

$$\delta x = x_0 + \mathrm{S}(x - x_c) + \mathrm{B}r^2 (x - x_c)$$

$$\delta y = y_0 + \mathrm{S}(y - y_c) + \mathrm{B}r^2 (y - y_c)$$

where B is the coefficient of the cubic radial distortion and S is the amount by which the scale parameter was distorted in the initial fit of the 26-inch measures to the 1.0m measures in order to absorb as much as possible of the cubic distortion. The parameters x_0, y_0, x_c , y_c , S , B were determined in a least squares fit solution from the residuals plotted in Fig. 1. Values obtained were

scale : $\mathrm{S} = (0.974 \pm 0.033) \times 10^{-3}$
cubic : $\mathrm{B} = (-2.092 \pm 0.068) \times 10^{-7}$

The coordinates determined for the optical aids were (495, 175) mm, with standard errors of 0.5mm, which compare well with the measured coordinates of the field centre quoted for the 1.0m plate of (497, 173) mm. The coefficient of the cubic term is in excellent agreement with Lowne's theoretical value. This value of the scale was used to give the linear term plotted in Fig. 3. The values obtained for the constant terms x_0 and y_0 are not important. These terms are. to allow for the possibility that in fitting the 26-inch measures to the uncorrected 1.0m measures the residuals would not be symmetrical about the optical axis of the 1.0m if the stars were not well-distributed about this point, The values obtained for y_0 and y_0 were small, about (0.002) mm.

Conclusions

It has been demonstrated that the expression derived by Lowne giving the radial distortion of the field of the 1.0m telescope is very accurate, and if this correction is applied to the measures of a 1.0m plate then the astrometric positions can be determined to 0.2 arcsec rms or better (since this rms value comes from the differences of two sets of measures). The distortion varies as the cube of the distance from the plate centre. If the measures are not corrected for the distortion then the resulting residuals from an astrometric reduction exhibit a pattern quite different from a cubic, as the solution for the scale of the plate absorbs most of the cubic variation. In order to apply the correction the coordinates of the optical axis must be known to an accuracy of a few mm. In the extreme case of a star near the edge of the plate (r = 80mm) an error of the coordinates of the optical axis of 1 mm will cause an error in the value of the correction of

$$2.05 \times 10^{-7} \times 3 \times 80^2 \times 1 \text{ mm} = 4 \text{ microns} = 0.1 \text{ arcsec}$$

It has been demonstrated that if the plate contains a large number of well-distributed reference stars with accurately-known positions then it is possible to solve for the coefficients of the cubic distortion term and the coordinates of the optical axis. The solution must be done simultaneously with the solution for the plate scale.

Reference

Harmer, C. W. F. and Wynne, C. G., 1976. A simple wide-field Cassegrain telescope, Mon. Not. R. Astr. Soc., 177, p25p.

Figures 1 and 2   Differences of measures between JKT and 26-inch

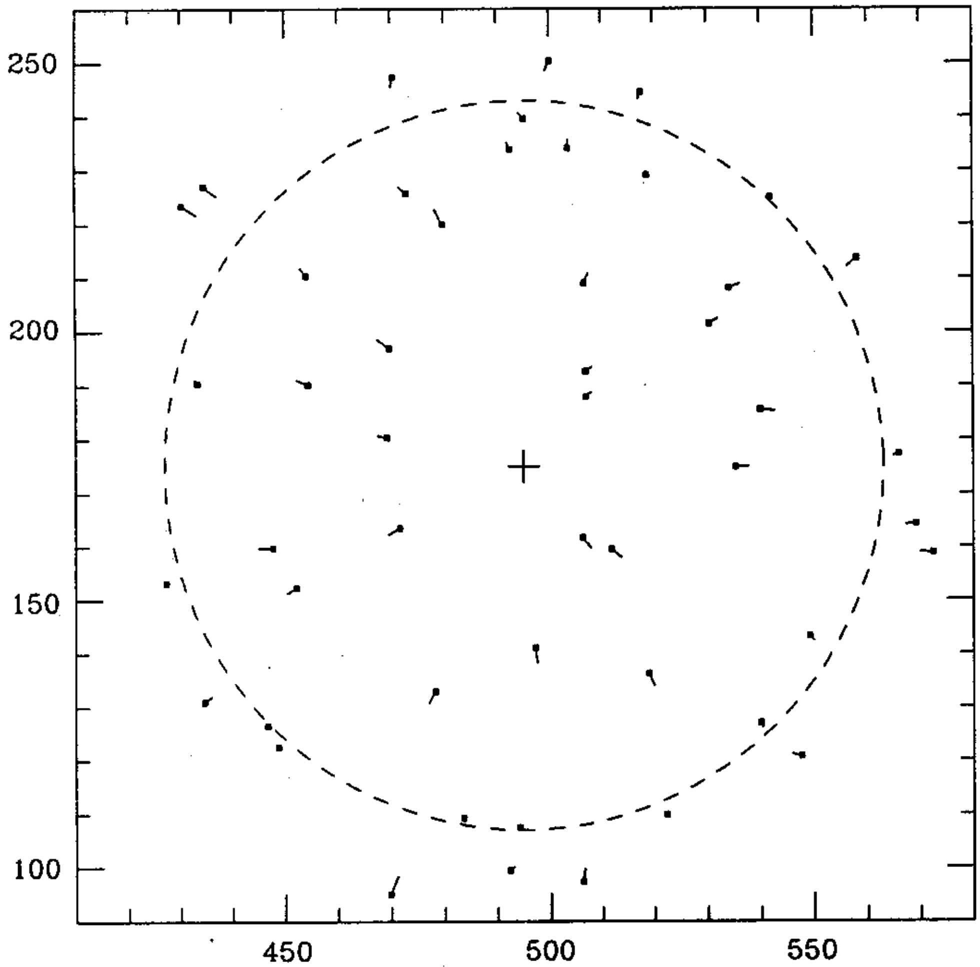

Figure 1 No radial correction applied

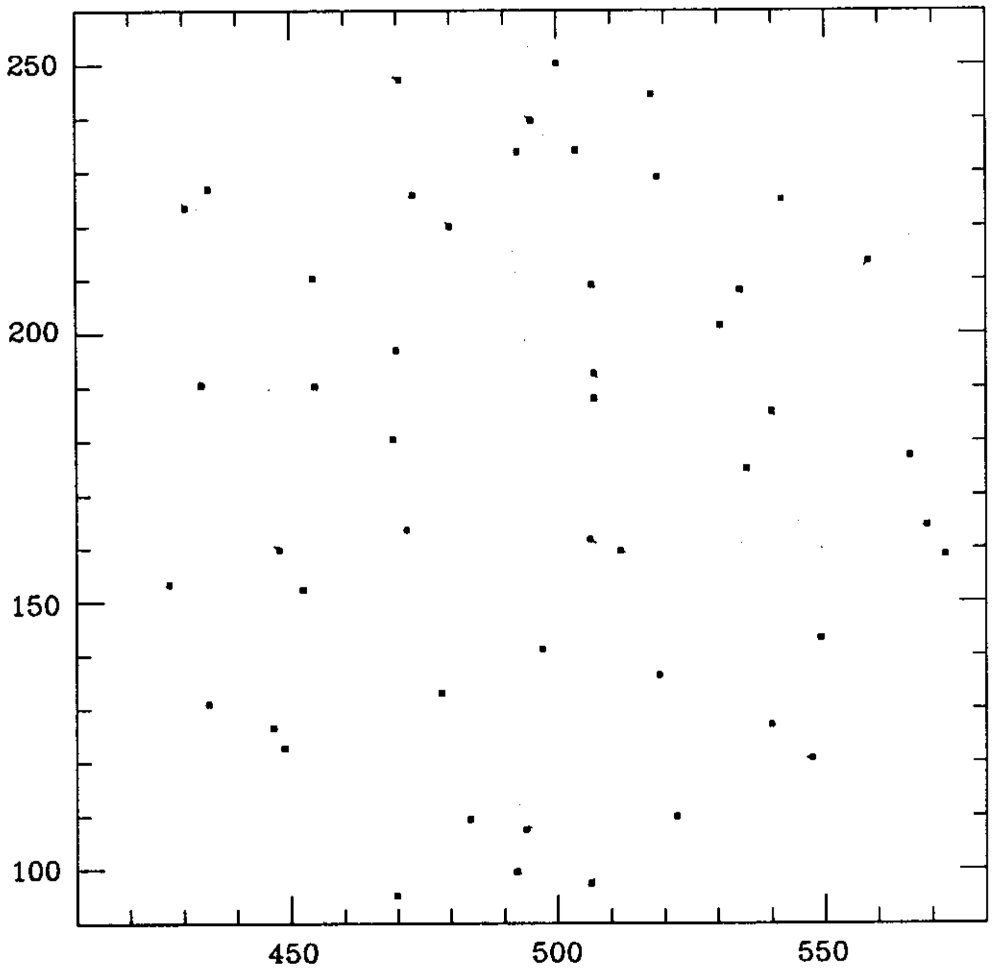

Figure 2 Radial distortion correction applied to JKT measures

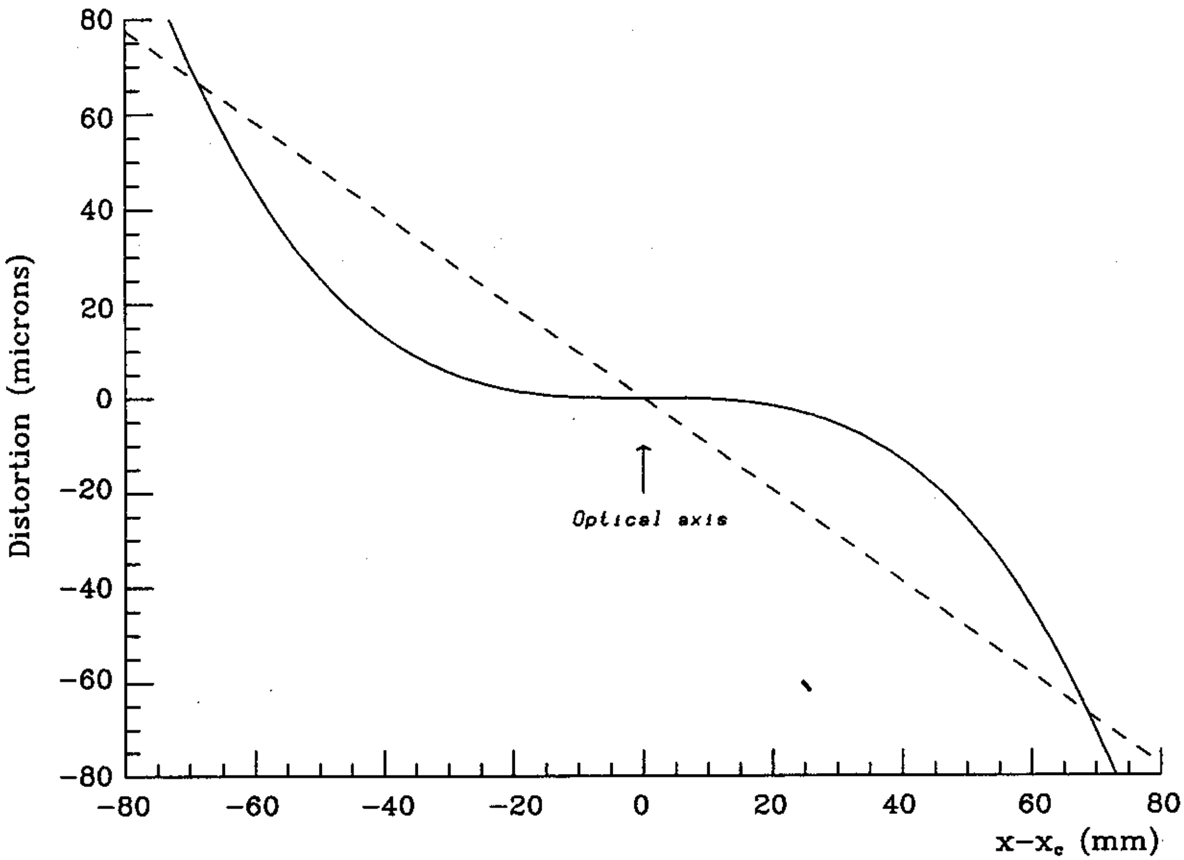

Figure 3 Effect of theoretical cubic distortion along the x-axis passing through the optical axis. Also plotted is best-fitting linear term

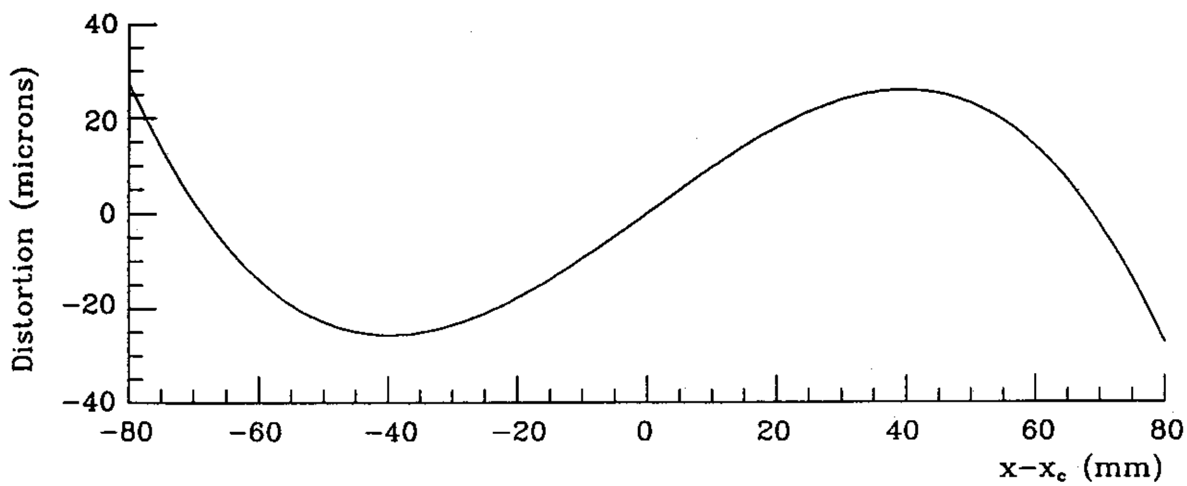

Figure 4 The pattern of the cubic distortion term along the x-axis after a best fitting linear term (or plate scale) has been removed