PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC
91:149-151, February 1979

ANAMORPHIC MAGNIFICATION OF GRATINC SPECTROGRAPHS—A REMINDER

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Received 1978 September 1

Attention is called to the long-known, but often ignored, property of a grating spectrograph to (de)magnify the slit width and length by different amounts. As an illustration, the "grating magnification" r is given as a function of grating tilt for the Ritchey-Chrétien spectrograph of the CTIO 4-meter telescope; we find that for large grating tilts the spectrograph slit can be opened to more than twice the usual width. This results in significantly more star light going down the slit in average seeing.

Key words: instrumentation—observing techniques—spectroscopy

Any optical system that produces different magnification along mutually perpendicular radii is called anamorphic. A grating spectrograph is just such a system, as was already known to Bowen (1952), in whose review articles (1955, 1962) many older references concerning grating theory can be found.

To briefly review the anamorphic property of a grating spectrograph, assume that the entrance aperture of the spectrograph is a slit of width W and length L aligned so that the length of its projected image lies perpendicular to the direction of the dispersion. This projected length ℓ at the plate is then

$$\ell = (f_{\rm cam}/f_{\rm coll})L \quad , \tag{1}$$

where $f_{\rm cam}$ and $f_{\rm coll}$ designate the focal lengths of the spectrograph camera and collimator, respectively. Since in most astronomical spectrographs $f_{\rm cam} < f_{\rm coll}$, $f_{\rm cam}/f_{\rm coll} < 1$ and the reciprocal of this ratio is often called the demagnification or reduction factor.

Whereas the above reduction factor for the slit length is well known and widely used, only few observers seem to be aware that a different reduction factor applies to the slit width. This difference is due to the fact that in the direction of dispersion the grating itself produces additional demagnification or magnification, depending on its tilt. To understand this, imagine the spectrograph slit illuminated by a source of monochromatic light of wavelength λ and follow the light path from the collimator via the grating to the camera, as sketched in Figure 1. In that figure, α represents the angle of an incident ray with respect to the grating normal (GN); β the angle of the diffracted ray; and t the tilt between the GN and the line bisecting the angle between the incident and diffracted rays. Note that β and t are counted positive if they lie on the same side of the GN as the incident ray, and negative otherwise. The angle Φ between incident and diffracted rays is also marked. Inspection of the figure shows that

$$\Phi = \alpha - \beta \quad , \tag{2}$$

$$t = \alpha - \Phi/2 = \beta + \Phi/2 \quad , \tag{3}$$

and, therefore,

$$\alpha = t + \Phi/2 \tag{4}$$

and

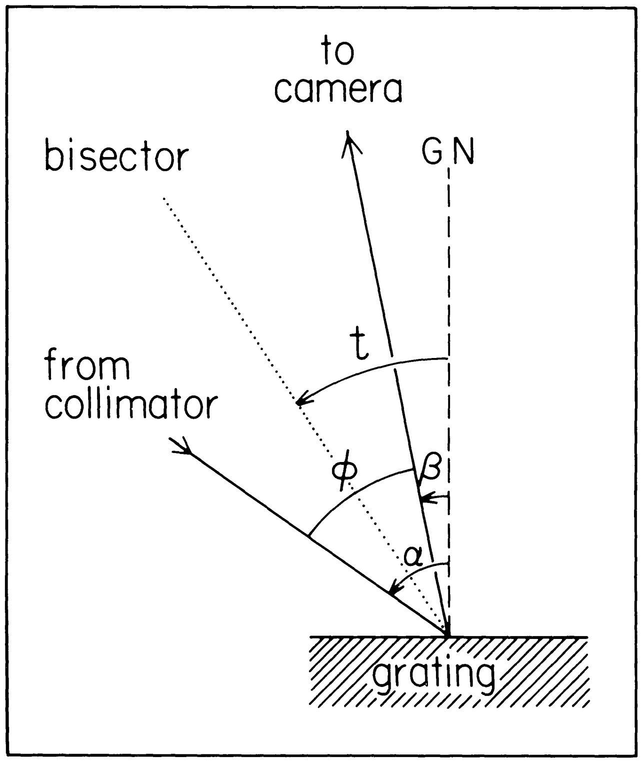

FIG. 1—Path of monochromatic light ray being diffracted by a grating. The angle of incidence α, angle of diffraction β, and grating tilt t are referred to the grating normal (GN). As usual in grating optics, angles on the incidence side of the grating normal are counted positive, those on the opposite side negative. In this figure, the angles α, β, and t are all positive.

*Cerro Tololo Inter-American Observatory is supported by the National Science Foundation under contract AST 74-04128.

$$\beta = t - \Phi/2 \quad . \tag{5}$$

Now consider a pair of rays which originate from opposite edges of the slit (i.e., from two points separated by W) and which hit the grating with incident angles differing by $\Delta\alpha$. After diffraction by the grating, the two rays form an angle $\Delta\beta \neq \Delta\alpha$, as can be seen immediately by differentiating the well-known grating equation,

$$\sin\alpha + \sin\beta = n\lambda/g \quad , \tag{6}$$

where n is the order of the diffracted spectrum and g the linear groove separation, for constant wavelength:

$$\cos\alpha \, d\alpha + \cos\beta \, d\beta = 0 \quad . \tag{7}$$

The resulting (de)magnification in the direction of dispersion due to the grating is then

$$\begin{aligned} r &= -d\beta/d\alpha = \cos\alpha/\cos\beta \\ &= \cos(t + \Phi/2)/\cos(t - \Phi/2) \quad , \end{aligned} \tag{8}$$

where the last form has been derived using equations (4) and (5), and normally is the most useful for practical evaluation. The overall (de)magnification of the spectrograph including the part due to the camera and collimator, results in a projected slit width w of

$$w = r(f_{\mathrm{cam}}/f_{\mathrm{coll}})\, W \quad . \tag{9}$$

The combined "reduction factor" $(r\, f_{\mathrm{cam}}/f_{\mathrm{coll}})^{-1}$ in the direction of dispersion obviously differs from that in the direction perpendicular to the dispersion if $r \neq 1$, which is the case if $|\alpha| \neq |\beta|$. This condition is met in most modern Cassegrain and coudé spectrographs.

The factor r was given no name by Bowen, who called it the "ratio of change in angle of camera beam in direction of dispersion to change in angle of collimator beam" (Bowen 1962). Recently, that factor has been quoted under various names, such as, somewhat inaccurately, "anamorphic magnification" (Chaffee and Schroeder 1976) and, more appropriately, "grating magnification" (Wheeler 1973). Note from equation (8) that the grating magnification is a function of the grating tilt; it takes the values $r = 1$ for $t = 0$, the zeroth-order tilt (which corresponds to $\alpha = -\beta$), and $r \lesseqgtr 1$ for $t \gtreqless 0$. Until the early 1960s, tilts used in stellar coudé and Cassegrain spectrographs were generally small, leading Bowen (1962) to remark that "r rarely departs appreciably from unity"; this probably explains the widespread neglect of r in the computation of slit widths by modern observers. On the other hand, tilts used in echelle spectrographs are generally so large that r is typically 0.4–0.7, a fact seemingly well known to users of these instruments.

It is the purpose of this brief note to emphasize that with the increased use of high-dispersion gratings in spectrographs with cameras of short focal length, especially in Cassegrain spectrographs equipped with image tubes, the grating magnification can no longer be neglected in the computation of slit widths from equation (9). To illustrate the point, Figure 2 shows r as a function of grating tilt for the Ritchey-Chrétien grating spectrograph of the CTIO 4-meter telescope. In this spectrograph, Φ is $\sim 46°$ by construction, and grating tilts up to about 45° are used (see the *CTIO Facilities Manual*), resulting in values of r as low as ~ 0.40. Some frequently chosen grating tilts and the corresponding r factors are indicated in Figure 2 by dots on the curve and labeled with the spectral line that appears centered and with the approximate linear reciprocal dispersion (in Å mm^{-1}, for the Singer camera

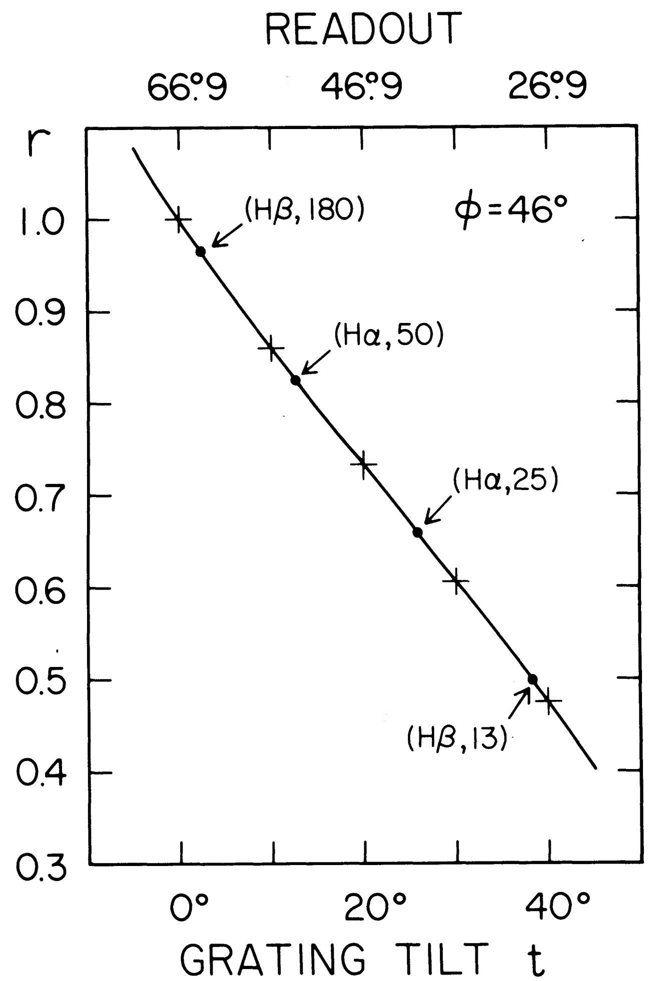

FIG. 2—"Grating magnification" r as a function of grating tilt t for the RC spectrograph of the CTIO 4-meter telescope. Four often-used tilts are indicated by dots on the curve and labeled with the resulting central spectral features and approximate linear reciprocal dispersions (in Å mm^{-1}, for the Singer camera plus Carnegie image tube). The grating-tilt readout of the spectrograph (= 66°9 − t) is given at the top of the figure for the convenience of users. With r taken from this graph, the projected slit width is $w = r\,(f_{\mathrm{cam}}/f_{\mathrm{coll}})\, W$, where $f_{\mathrm{cam}}/f_{\mathrm{coll}}$ = 0.197, 0.258, and $\sim$ 0.237 for the Schmidt camera, the Singer camera, and the Singer camera plus image tube, respectively, and W is the slit width of the spectrograph.

with Carnegie image tube).

Take the case of an observer wishing to use the 1200 line mm^{-1} grating in second order to observe at Hβ with a linear reciprocal dispersion of 13 Å mm^{-1} ($t = 38\overset{\circ}{.}45$, see Fig. 2). If he aims for a given projected slit width at the plate (normally chosen to match the plate or image-tube resolution), equation (9) tells him that he can open the spectrograph slit 2.0 times more than if he ignored the factor r. This means that to achieve $\sim$ 30 μm resolution at the plate with the Singer camera plus Carnegie image tube ($f_{cam}/f_{coll} = 0.258$, image-tube demagnification about 8%), he can open the spectrograph slit to $\sim$ 250 μm $\approx$ 1.6 arc sec, rather than half that value if he ignored the r factor. In average seeing, this results in a significant increase of star light going down the slit.

Of course, this increased throughput of light at a given resolution is one of the main reasons why spectrographs are designed for use with positive grating tilts.

I thank Linda Schweizer for the reminder and Nolan Walborn for helpful discussions and comments.

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