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SOFTWARE FOR MULTI-SCALE IMAGE ANALYSIS: THE NORMALIZED OPTIMIZED ANISOTROPIC WAVELET COEFFICIENT METHOD

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Abstract—The two-dimensional Anisotropic Wavelet Transform aims to decipher images in which distributions are combined at different scales. Based on the Optimized Anisotropic Wavelet Coefficient method (OAWC), we present two C^{++} programs which enable multi-scale analysis and discrimination of objects, or groups of objects, depending on their area, shape ratio, orientation, and location. These programs allow the identification of the different levels of organization that are present in an image (objects, clusters, alignments of clusters), and the quantification of their sizes, shape ratios, and orientations. The programs run on a standard IBM-PC. A complete analysis, performed on a constructed image, is presented as an example for its graphical results, and illustrates the potential of quantifying the anisotropies of orientation, shape, and spatial distribution of organized structures at different scales. © 1997 Elsevier Science Ltd

Key Words: Image processing, Multi-scale analysis, Two-dimensional Anisotropic Wavelet Transform, Spatial distribution.

INTRODUCTION

Physical processes act simultaneously at different scales, and lead to complex spatial distributions. Sorting out each part or level of the spatial organization, according to its characteristic scale, may be an effective way of analysing complex systems. Grossman and Morlet (1984) have developed an innovative numerical formalism, the Wavelet Transform (WT), in order to decorrelate the one-dimensional signal of seismic-reflexion data. In two dimensions, a number of developments in mathematics and theoretical physics have investigated multi-scale behaviour of a system applying the WT formalism. The WT is now applied to several fields, such as meteorology (e.g. Hagelberg and Helland, 1995) and astrophysics (e.g. Escalera and MacGillivray, 1995), but still rarely in the geosciences (Ouillon, Castaing, and Sornette, 1996), despite the numerous potential examples of complex (multi-scale) organizations (fault patterns, topography, seismic data, and many other examples).

Rocks are complex systems in which the superposition of different physical processes such as crystallization, deformation, and fluid circulation, lead to fabric development. Automatic determination of the Shape Preferred Orientation (SPO) of crystals is now a popular method for analysing the petrofabrics in rocks (e.g. Allard and Benn, 1989; Launeau, Bouchez, and Benn, 1990; among others). However, this analysis ignores the two-dimensional multi-scale distribution of objects, such as clusters of minerals and alignments of clusters. The scope of this paper is to promote a new way of multi-scale investigation using the two-dimensional Anisotropic Wavelet Transform (2D-AWT).

For that purpose, we present two programs enabling the user to detect the different levels of organization of an image, and to quantify the geometry of the detected entities in terms of area, shape ratio, orientation, and location. The programs have been written in C^{++} and run on a standard IBM-PC. An Intel Pentium processor and at least 8 Mbytes of Ram are recommended. Programs are compatible with Microsoft Windows 3.11 and Windows 95. Input is via user-friendly dialogue boxes, graphical outputs of the results are displayed on the screen, and output as files. The programs are publicly available by anonymous FTP from IAMG.ORG.

The program WaveCalc calculates a set of two-dimensional-AWT, with an automatic and objective selection of the information. For the computation of the WT, subroutine FFT (Elliott and Rao, 1982) is used, performing the calculations of both the direct and inverse FFT. The companion program WaveSea is essentially a graphical interface allowing the visualization of the results obtained from

WaveCalc. WaveSea helps the user to select the most significant features at each scale, and gives the geometric parameters of the detected structures for each level of organization, performing basic functions such as thresholding, and building of histograms or orientation rose diagrams. Information obtained from the software permits the signal at all scales to be quantified and consequently facilitates the understanding of the processes which underlie the fabric under study.

WAVELET TRANSFORM: FROM THEORY TO PRACTICE

The potential of wavelets applied to image processing has been pointed out by Antoine and others (1992), and a number of theoretical developments clearly demonstrate the wide domain of application of the Wavelet Transform (WT) formalism (Meyer, 1992). In the applications presented so far, most authors have used, for obvious practical reasons, the two-dimensional isotropic WT. Calculation of the two-dimensional-anisotropic WT (two-dimensional-AWT) requires a much longer computer time and generates a large quantity of data which need to be synthetized. Recently, Ouillon, Castaing, and Sornette (1996) have studied the multi-scale organization of joints and faults, using an efficient method called the Optimized Anisotropic Wavelet Coefficient method (OAWC) that selects the pertinent information of an image at all scales. Our approach is based on the latter method developed by Ouillon, Sornette, and Castaing (1995) and carries out the complete multi-scale analysis of an image with an accurate description of the size, shape, orientation, and spatial distribution of the objects.

Theory

In order to show how the two-dimensional-AWT detects the dominant structure at a given scale, the mathematical formalism of the Wavelet Transform is summarized here. The two-dimensional continuous WT helps in deciphering signals in which information carried by different spatial wavelengths or scales is combined using a battery of filters called wavelets. The filters are derived from a single function called the mother function Ψ_σ. In the present work, we use the "Anisotropic Mexican Hat", which comes from a second derivative of a Gaussian function. Due to its anisotropy, characterized by its shape ratio σ, the filter allows detection of shape anisotropy and singularities in all directions of the image plane. Its equation is given by:

$$\Psi_\sigma(\vec{x}) = \Psi_\sigma(x,y) = \left(2 - \frac{x^2}{\sigma^2} - y^2\right) e^{-\frac{1}{2}\left(\frac{x^2}{\sigma^2} + y^2\right)} \qquad (1)$$

where x and y are the Cartesian coordinates of each pixel of the image.

Taking a binary image, the image can be represented by the function $I(\vec{x})$, $I(\vec{x}) = 1$ for pixels belonging to the analysed objects and by $I(\vec{x}) = 0$, otherwise. The WT of $I(\vec{x})$ is a convolution (C_I) with an analysing wavelet $\Psi_\sigma(\vec{x},a,\theta)$:

$$C_I(\vec{x},a,\sigma,\theta) = \Psi_\sigma(\vec{x},a,\theta) \bigotimes I(\vec{x}), \qquad (2)$$

where a and θ represent the resolution (short axis of the wavelet) and the azimuth of the long axis of the wavelet, respectively. For each analysing wavelet, the signal is transformed into a set of coefficients C_I by the convolution. This set is called the wavelet image and is represented by a coefficient map. Thus, at a given position $\vec{x} = \vec{b}$ in the signal, the wavelet coefficient $C_I(\vec{b},a,\sigma,\theta)$ is given by:

$$C_I(\vec{b},a,\sigma,\theta) = \frac{1}{K_{\Psi_\sigma}} a^{-1} \int \Psi_\sigma\left(R_\theta^{-1}\left(\frac{\vec{x}-\vec{b}}{a}\right)\right) I(\vec{x}) \mathrm{d}\vec{x} \qquad (3)$$

where R_θ is the counterclockwise rotation operator of angle θ with respect to the Ox axis, which allows the analysing wavelet to be rotated. The scale parameter a allows the dilation (large a value) or contraction (small a value) of the wavelet, and thus controls the resolution of the image analysis. The normalizing factor K_{Ψ_σ} is defined following the admissibility condition by:

$$K_{\Psi_\sigma} = \int |\Psi_\sigma^*(\vec{k})|^2 |\vec{k}|^{-2} \mathrm{d}\vec{k} \qquad (4)$$

where Ψ_σ^* is the Fourier transform of Ψ_σ.

Properties

A wavelet is a function that obeys the following four conditions detailed by Antoine and others (1992):

(1) The mother function Ψ_σ must be well localized in the space domain ($\vec{x}$) and in the spatial frequency domain ($\vec{k}$);

(2) Ψ_σ is continuous and differentiable (admissibility condition); as an important consequence it always has a null mean value;

(3) The analysing filter $\Psi_\sigma(\vec{x},a,\theta)$ is naturally bound and invariant by translation: thus, it is particularly suited for a local analysis; and

(4) $\Psi_\sigma(\vec{x},a,\theta)$ is also invariant by dilation/contraction, thus allowing a multi-scale analysis.

In practice

Due to its intrinsically multi-scale and local properties, the WT is particularly useful for the detection of small structures superimposed on the

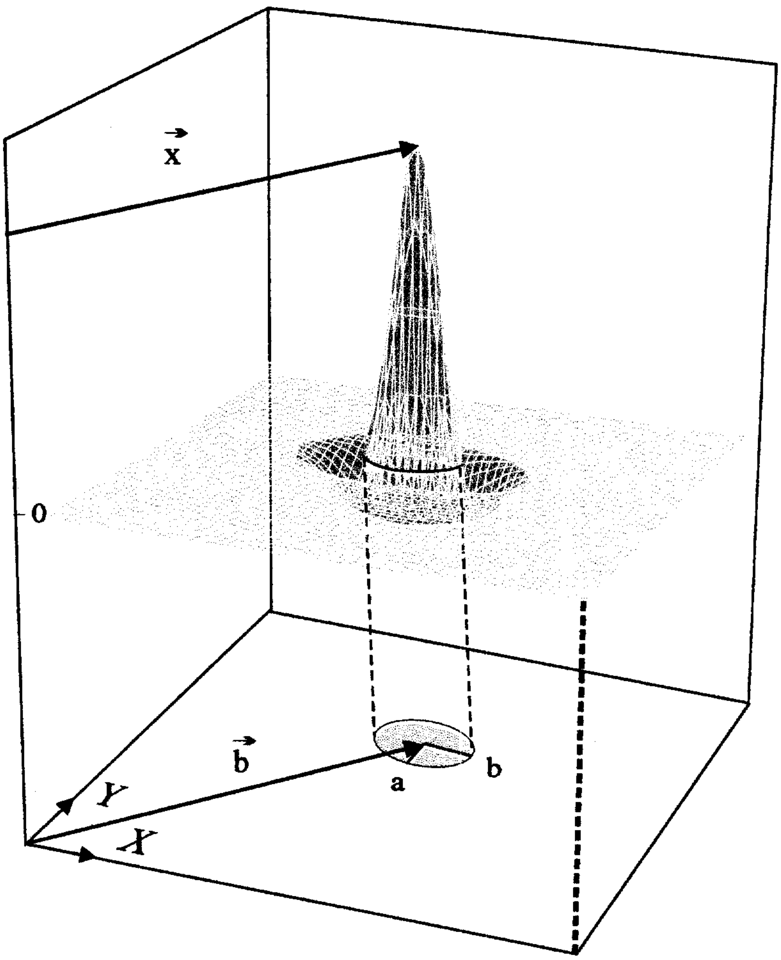

Figure 1. Ideal match between wavelet and object: zero contour of Anisotropic Mexican Hat fits perfectly with object contour. The filter $\Psi_\sigma(\vec{x},a,\theta)$ is discretized in $2^6 \times 2^6$ mesh world. Resolution $a = 2$, shape ratio $\sigma = 2$, orientation $\theta = 0°$, location $\vec{x}$ (32, 32). Corresponding detected entity: short axis $a = 2$, long axis $b = 4$, shape ratio $b/a = 2$, elongation $\theta = 0°$, location $\vec{b}$ (32, 32).

gradient of larger ones. A constant signal produces null coefficients (= admissibility condition). In contrast, if the signal presents irregularities, the wavelet reacts by producing non-null coefficients. The negative coefficients underline the edges of objects. The weak coefficients reflect a poor match between the analysing filter and the local content of the image. Conversely, the strongest coefficients point out the best match, in terms of scale, location, shape ratio, and orientation, between the analysing wavelet and the object. A given filter $\Psi_\sigma(\vec{x},a,\theta)$ detects only entities that have the identical geometrical parameters (Fig. 1). To detect all the organized structures of an image, each pixel of the two-dimensional-signal will be analysed using a set of different a, θ, σ values.

WAVECALC PROGRAM

The WT of a ($2^8 \times 2^8$) binary image requires 3 h of calculation (Pentium 90 Mhz) to obtain a complete analysis. Its application to an artificial example is given in Figures 2 and 3. In order to avoid operator time during computation, the program WaveCalc operates directly from ASCII parameter files. Intervention of the user is required to input the following parameters from the keyboard.

1. the number of scale analyses (N) and, for each analysis,
2. the binary image file name (WINDOWS BITMAP, TIFF, RASTER),
3. the image width (W), height (H);
4. the resolution or scale parameter a, which controls the multi-scale analysis;
5. the limits of integration and their increments (b_min, b_max, step_b), which control the shape anisotropy; and
6. the variation domain and increment of orientation which control the angular exploration (θ_{min}, θ_{max}, step_θ) of the image.

The program computes the WT of the image for each $\Psi_\sigma(a,\vec{b},\theta)$ filter.

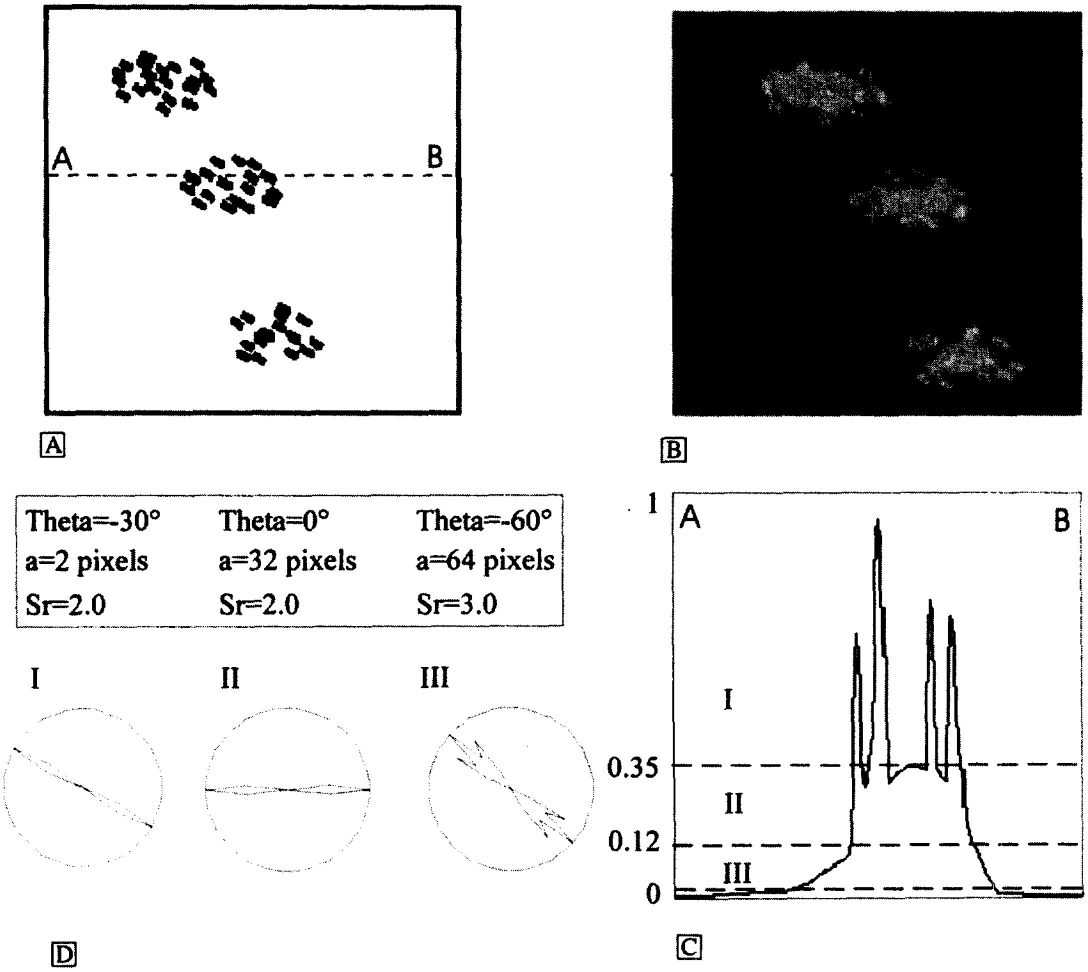

Figure 2. Study of synthetic example. (A) Constructed ($2^8 \times 2^8$) binary image composed of 52 rectangles of area = 45 pixels, shape ratio = 2 and orientation = −30° themselves forming three clusters elongated at 0°, forming themselves an alignment at −60°. (B) Synthetic NOAWC map obtained from 2, 32 and 64 pixels resolution. (C) A–B cross-section: highest coefficients give centre of gravity of individual objects of image A; levels around 0.3 (pale grey in B) correspond to grain clusters. Coefficients around 0.1 (in dark grey) point to alignment formed by clusters; (D) Geometrical parameters obtained after thresholding with WaveSea: (I) individual object parameters; (II) cluster parameters; and (III) alignment of cluster parameters.

Although the WT is perfectly well-defined in real space, frequency considerations are useful. The Fourier Transforms (FT) operator allows an efficient calculation of the WT. The numerical implementation of the direct and inverse FT is simple using the Fast Fourier Transform (FFT) algorithms (Elliott and Rao, 1982). Note that, for the application of the FFT algorithms, the image must be a $2^n \times 2^n$ size matrix (n = integer). For reasons of calculation time, memory allocation, and disk space, the maximal size of the image in this code is limited to $2^9 \times 2^9$ pixels. The procedure for calculating the WT is the following:

1. read the binary image matrix;
2. compute the two-dimensional discrete FFT of the image, and the analysing wavelet, using the theorem of separability;
3. following the convolution theorem, determine the FT of the WT by multiplying the FT of the image with the FT of the analysing wavelet; and
4. determine the coefficient map of the image by applying the inverse FFT.

Using filters with different σ values necessitates the use of different mother functions. In order to obtain a significant calibration, the results are normalized as follows: at each point of the image, the calculated coefficients are divided by their theoretical maximum values obtained when the match between the filter and the corresponding entity is perfect. A linear scaling from −1 to 1 is therefore obtained. The highest value corresponds to the centre of gravity of the entities detected; null values correspond to a constant gradient, and the smallest values indicate the edges of the structures. This

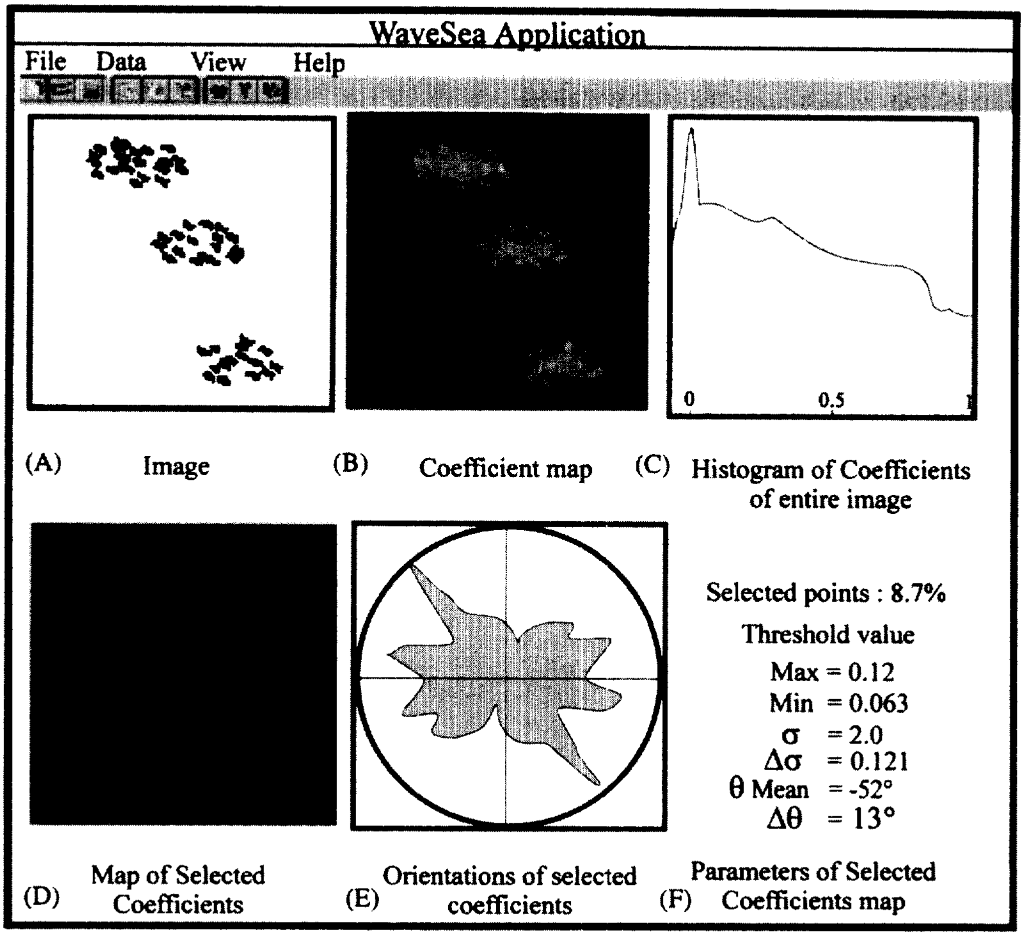

Figure 3. Graphical interface WaveSea showing original image (upper left), Normalized Optimized Anisotropic Wavelet Coefficients (NOAWC) map of resolution $a = 32$ pixels, histogram of normalized coefficients and information on image (upper right). For interactively determined threshold (0.3–0.4), corresponding selected NOAWC map (lower right), orientation rose (mean −52°; shape ratio parameter Sr = 2.00) is plotted.

scaling of the coefficients reinforces the image contrast and facilitates the interpretation. Repetition of this procedure for each filter creates a large volume of data. To synthesize these data, a method is proposed extended from the OAWC algorithm of Ouillon, Sornette, and Castaing (1995). As already mentioned, the value of the coefficient reflects the local match between the filter and the signal. To select the meaningful features at each point of the signal, we choose the local optimum filter characterized by the maximal wavelet coefficient. The maximized normalized coefficients are selected from the coefficient maps derived from all the (σ, θ) couples, and for a given resolution a, at each pixel of the image. Hence, at resolution a, a Normalized Optimum Anisotropic Wavelet Coefficient map (NOAWC map) is derived and reflects the match between the given filters and the image (Fig. 1). During this step, the parameters corresponding to the local optimum filter (σ, θ) are stored. Hence, in output, a normalized coefficient file, and the associated shape ratio and orientation files are given. This procedure is repeated for all the selected resolutions. Finally, the program selects among all the NOAWC maps corresponding to each resolution, and for each point, the optimum wavelet coefficient and its associated parameters (resolution, position, orientation, and shape ratio). The "synthesized" NOAWC map ("single-interest image") is finally visualized on a screen using coefficients that are scaled and transformed to 8-bit integers covering the 0 (black)–255 (white) range (Fig. 2).

The volume of data remains substantial in spite of their significant reduction. For an image size of $2^n \times 2^n$ pixels, there are 2^{2n} coefficients, 2^{2n} resolutions, 2^{2n} shape ratios, and 2^{2n} angles. Hence, for $n = 8$, the total number of figures to handle is 4×2^{16}! The WaveSea program has been developed to detect the significant organized structures and to extract the corresponding parameters.

WAVESEA PROGRAM

This program is a graphical interface which (1) gives access to all the parameters of the original image (Fig. 3A), as well as to the NOAWC maps (Fig. 3B); (2) allows recognition of the meaningful features; and (3) retains only the significant set of parameters that describes best the image structures. In the coefficient map of an image, every scale of organization is detected, but with different values of the coefficients. These levels depend on the original image, but also on the resolution. To guide the user to choose the level that characterizes best each structure [Figure 2(B) and (C)], the histogram of the NOAW Coefficient map is presented [Figure 3(C)]. From it, the different modes of the NOAWC coefficients are easily recognized, and the minimum and maximum thresholds are chosen interactively. The parameters (resolution, location, shape ratio, orientation) corresponding to the selected local optimum filters are instantaneously extracted from the set of data.

Finally, the map of the selected NOAWC is plotted (Fig. 3D), and the corresponding orientations are presented in a rose diagram. General information, such as the percentage of objects in the original image, percentage of selected pixels in the NOAWC map, and statistical parameters (mean and standard deviation of orientation and shape ratio data), are also presented (Fig. 3F). The selected data may be saved in a standard text file directly usable in a spreadsheet.

GEOLOGICAL EXAMPLE

The multi-scale organization of a Variscan pluton (Sidobre granite, Montagne Noire, France) has been studied using the NOAWC method (Fig. 4). A 70 cm × 100 cm sample was cut in the *XZ* plane, that is parallel to the magmatic stretching direction (*X*) and perpendicular to the magmatic flattening plane (Figs 4A and 4B). The multi-scale mineral organization (or fabric) analysis was performed on the K-feldspar megacrystals represented in black in Fig. 4A).

In the resultant binary image (256 × 256 pixels), a nonuniform spatial distribution shows a complex rock crystallization and deformation history. In this study, the resolutions are chosen as multiples of 2 ($a = 2n$, with $n = 1,2,...,10$). For each resolution, the integration scale σa varies in between the sampling scale (a) and the image size itself. For each couple ($a,\sigma a$), the azimuth θ varies from 0° to 180°, by 10° steps. Two significant levels of mineral organization have been observed, for $a = 2$ and $a = 10$ pixels. For the other values of a, magnitudes

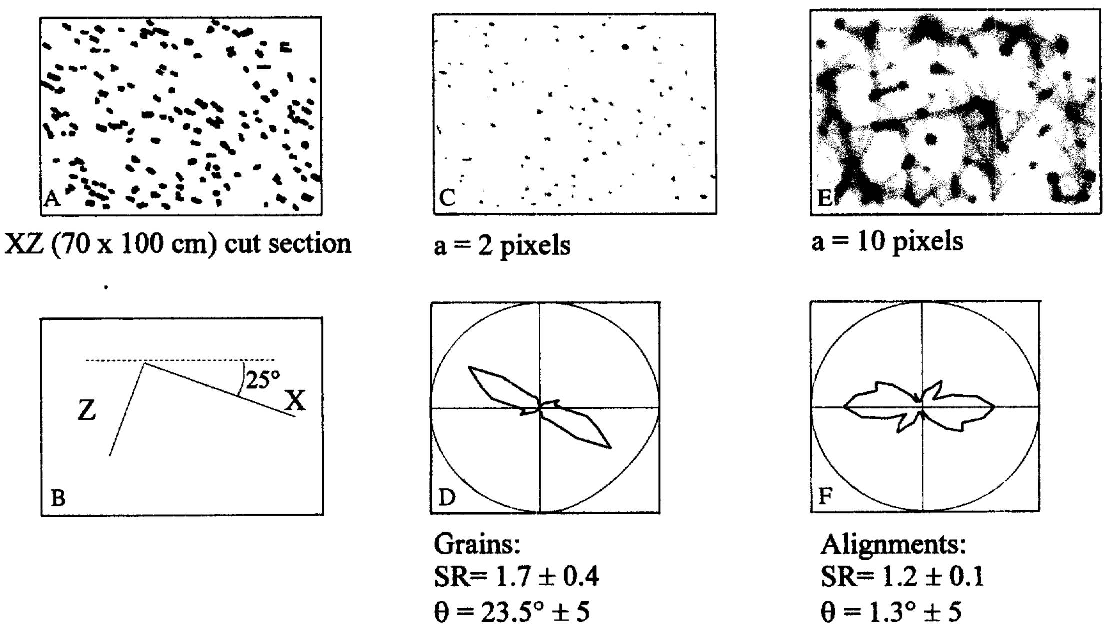

Figure 4. Multi-scale investigation of (70 cm × 100 cm) *XZ* section of K-feldspar megacrysts of Sidobre granite (France). Normalized Optimized Anisotropic Wavelet Coefficients (NOAWC) map obtained for two resolutions: $a = 2$ pixels (0.8 cm) and $a = 10$ pixels (4 cm). Orientation rose diagram at mineral scale: selected coefficient range between 0.75 and 1 identifies megacryst characteristics. They are subparallel to *X* direction and indicate magmatic flow. Orientation rose diagram at aggregate scale: selected coefficient range between 0.35 and 0.45 identifies cluster of megacrystals, with principal mineral alignments (around 0°) orientated at −50° and 30°, representing magmatic shearing. (Source: Gaillot and others, 1997).

of the wavelet coefficients are weak, denoting a lack of significant organization at these scales.

For a resolution of $a = 2$ pixels, corresponding to crystal sizes (≈1 cm), a strong orientation at 23.5°, has been indicated (Figs 4C and 4D). This orientation is the same as the one detected by Darrozes and others (1994) using the intercept counting method of Launeau, Bouchez, and Benn (1990). For $a = 10$ pixels, a clearly defined cellular organization (Figs 4E and 4F) underlined by grain alignments at −55°, 0°, and 30° has been detected (see Gaillot and others, 1997).

In this example, detection for a resolution of $a = 2$ pixels reveals the crystal organization at the scale of the grains and the magmatic flow direction. This aspect of rock-texture analysis may be worked out using other fabric techniques. However, the NOAWC method is unique in quantifying and mapping crystal organizations at larger resolutions ($a = 10$ pixels). These large-scale organizations are fundamental in understanding the rheological behaviour of magmatic rocks, for instance by tracing the solid versus liquid partition in the rock, as exemplified by the cellular organization evident in Figure 4A. Hence, the NOAWC approach constitutes a fundamental contribution for rock fabric analysis.

SUMMARY

The multi-scale analysis based on the Wavelet Transform (WT) is presented in this paper in a complete and simple form. We have also given the mathematical background of the WT. In order to undertake a complete multi-scale analysis of a binary image of maximum size of $2^9 \times 2^9$ pixels, the C^{++} programs WaveCalc and WaveSea have been developed. WT computation, selection of data and calculation of geometrical parameters of detected structures are given in detail. The artificial example illustrates how the anisotropies of shape and orientation, as well as the spatial distribution of the objects can be quantified at different scales. These programs have also been applied to a geological example to determine a rock fabric. The WT detected and characterized accurately the different levels of mineral organization, from grains to alignments or clusters of grains, valuable information for studying the processes acting at different scales. Finally, since the NOAWC approach is applicable to any two-dimensional object distribution, it may be used in many other fields, such as fluid inclusions in crystals, epicentral distribution of seismic events, and distributions of galaxies.

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