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Purpose To determine the amount of vignetting of M1 mirror is produced by M2 baffle

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## 1. Introduction

The William Herschel Telescope (WHT) has a parabolic primary mirror (approximately 4.2 m $\mathrm{f} / 2.5$ ), a classical Cassegrain and two Nasmyth focal stations at $\mathrm{f} / 11$. These $\mathrm{f} / 11$ stations are all fed by the same secondary mirror, named M2 in the following diagram (Figure 1):


Figure 1 - WHT schematic

The size of the M2 baffle is a design parameter of the telescope, its diameter is determined by the size of the prime focus support structure, but we suspect the length of this baffle is not adequate, producing extra vignetting at the primary mirror. This document describes and discusses the computations and measurements done to confirm this, and to propose corrective action.

## 2. Initial data and measurements

Most of the data used in the calculations have been extracted from the ING "Observer's Guide, 1995" that can be found at http://www.ing.iac.es/Astronomy/observing/manuals/man_gen.html, like the primary mirror diameter and its focal length; some other data was obtained from the telescope Zemax models, like the distance between the centers of M1 and M2, and finally direct measurements to determine the size and position of the M2 baffle; some of these measurements are approximate to the order of a few millimeters, sometimes because of the mechanical tolerances of the pieces measured and sometimes because of the inaccessibility of the reference points.

In the calculations we use:

- $D=4.180 \mathrm{~m}$, working diameter of M1. See table 2.1 Observer's Guide.
- $f=10.4396 \mathrm{~m}$, focal length of M1. See table 2.1 Observer's Guide.
- F-number= 10.94 at Cassegrain focus. See table 2.2 Observer's Guide.
- Distance M1-M2=8.035 m, several Zemax models
- Baffle internal diameter $D_{b}=1.210 \mathrm{~m}$, measured
- Distance between baffle edge to M 2 support ring $=0.660 \mathrm{~m}$, measured
- Focal length of $M 2=-3.11 \mathrm{~m}, \mathrm{R}_{2}=6.23$
- m radius of curvature, computed
- $D_{M 2}=1.001 \mathrm{~m}$, diameter of M2. See table 2.1 Observer's Guide.
- Height of M 2 centre with respect to M 2 border $\mathrm{h}=0.020 \mathrm{~m}$, computed

All the measurements have been double-checked and refined using a BOSCH laser distance meter; for the distance between the baffle edge and the support ring of M2 we obtained: 0.659 m , $0.657 \mathrm{~m}, 0.658 \mathrm{~m}$ and 0.659 m , measured respectively at the right side, down, left and up sides respectively. From this support ring to the border of the M2 mirror we measured in the same relative positions: 13.2, $8.7,9$ and 11 mm respectively.

The M2 baffle is made of 2 mm steel sheet, with two joints along the axis and reenforced rings to keep the circular shape. Measurements of the internal diameter of the baffle at several positions gave: $1.2075 \mathrm{~m}, 1.211 \mathrm{~m}, 1.211 \mathrm{~m}, 1.215 \mathrm{~m}, 1.212 \mathrm{~m}$ and 1.213 m ; which are under the expected mechanical tolerances.

Measuring the distance (on axis) between M1 and M2, is obviously impossible because the centre of M1 is inaccessible, so we measured off axis first measuring from a reference point located about 0.5 m off axis, first to M1 and from the same reference point to the support ring of M2, obtaining: $1.955 \mathrm{~m}+6.084 \mathrm{~m}=8.039 \mathrm{~m}$, which considering that we have to subtract the height of M2 with respect to its support ring and add the height of M1 at 0.5 m of its axis, it is in concordance with the figure obtained from the Zemax model.

## 3. Computing the vignetting for axial rays

Given that the M1 mirror is parabolic, parallel rays on axis will be reflected at the mirror surface and cross the optical axis at a distance of the mirror focal length, in the paraxial approximation. We call $\boldsymbol{\alpha}$ the angle of the ray reflected at the M1 border with respect to the axis, and $\boldsymbol{\beta}$ to the angle of the ray connecting the focus of M1 with the border of the baffle.

Note that given that M2 is in the way of these rays they are reflected by M2, neither of these rays actually reaches the focus, but of course the focus point is the adequate reference point (in optics this focus point is where the so called virtual image of a star on axis is formed by M1).

In the figure 2 you can see a diagram with the geometry of these rays and the positions of M1 (cyan), M2 (magenta) and the baffle (blue). The ray in red colour determines the angle $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ is determined by the green ray. Finally X and Y axis represent the distance from M1 centre on axis and perpendicular to the telescope axis respectively.

WHT ray-traces vignetted by M2 baffle


Figure 2 - WHT rays diagram ( $X$ and $Y$ have different scales)

Using this reference frame the equation of the shape of M1 surface is:

$$
y^{2}=4 \cdot f \cdot x
$$

So the angle of a reflected ray is a function to the distance to the axis ' $y$ ':

$$
\tan a=\frac{y}{f-x}
$$

and then

$$
\tan a=\frac{y}{f-\frac{y^{2}}{4 \cdot f}}
$$

We obtain the limit angle $\boldsymbol{\alpha}$ when $y=D / 2$, and with it we can compute the effective diameter of M2 used and we can also compute the distance at which the limit ray cuts the baffle cylinder, respectively:

$$
\begin{gathered}
\text { EffectiveDiameter }_{M 2}=2 \cdot d \cdot \tan (\boldsymbol{\alpha}) \\
\qquad \text { Xcut }_{b}=f-\frac{D_{b}}{\tan (\boldsymbol{\alpha})}
\end{gathered}
$$

where d is the distance from the focus to the centre of M 2, Xcut $_{b}$ is the abscissa where the baffle is crossed by the limiting ray, and $D_{b}$ is the internal diameter of the baffle. The difference Xcut $_{b}-X_{b}$ (where $X_{b}$ is defined below) gives us the amount we should 'cut' from the edge of the baffle to avoid the vignetting on axis. See in figure 3, the cut is the distance between the red ray cutting the baffle and edge ray (green):


Figure 3 - Detail of light rays close to the baffle edge

Given the fact that the actual position of the baffle edge can only be measured referred to the M2 support ring, and the position of this mirror is known at the axis, we have to compute the height of the centre of M1 with respect to its border, its diameter being known. In first approximation we can consider the radius of M2 constant, i.e. approximately spherical, and then this height is:

$$
h=\left(\mathrm{R}_{2}-\sqrt{\left({R_{2}}^{2}-\left(\frac{\mathrm{D}_{\mathrm{M} 2}}{2}\right)^{2}\right)}\right)
$$

We can now compute the abscissa of the edge of the baffle as:

$$
X_{b}=f-\left(d+\text { distance }\left(\text { Baffle }-M 2_{\text {ring }}\right)-h\right)
$$

and the angle connecting the baffle edge and the focus of M1 is:

$$
\tan (\boldsymbol{\beta})=\frac{D_{b} / 2}{\mathrm{f}-\mathrm{X}_{\mathrm{b}}}
$$

so to compute the semi-radius of the unvignetted part of M1 we solve the equation:

$$
y^{2} \cdot \frac{\tan (\boldsymbol{\beta})}{4 \cdot f}+y-f \cdot \tan (\boldsymbol{\beta})=0
$$

the solution yields:

$$
y_{\text {limit }}=\frac{-1 \pm \sqrt{1+\tan (\boldsymbol{\beta})^{2}}}{\frac{\tan (\boldsymbol{\beta})}{2 \cdot f}}
$$

Given the geometry, we are only interested in the positive solution, which gives the diameter of the area not vignetted of M1:

$$
D n v_{M 1}=2 \cdot y_{\text {limit }}
$$

Considering that the potential light collecting area of M1 is its whole working diameter minus the area of the central obscuration $\left(4 \cdot \pi \cdot\left(D_{b} / 2\right)^{2}\right)$, the percentage of light losses due to the M2 baffle vignetting for a star on axis can be computed as:

$$
\text { Loss }=100 \cdot\left(1-\frac{D n v_{M 1}^{2}-D_{b}{ }^{2}}{D_{M 1}^{2}-D_{b}{ }^{2}}\right)
$$

Note: the correct value to use here for the diameter of the central obscuration is the external diameter of the baffle, not the internal one that we have used, as well as take into account the vanes; but the differences are not very significant.

## 4. Computing the vignetting for a finite field of view

The simplest approach to solve the problem is to consider that every small surface element of the primary mirror is not reflecting a single light ray on axis as considered in section 3, but a light cone whose axis is the ray crossing the focus of M1 (as before) and the cone angle is the field of view (FOV) of interest. So to compute the diameter not vignetted even partially, the limit angle $\boldsymbol{\beta}$ in the worst case is reduced by FOV/2 using the same equation to compute $y_{\text {limit }}$. This new limit is smaller than before and accounts for the area of M1 for which the axial ray is not vignetted but still some area of the FOV is partially vignetted.

$$
y_{\text {limit__partial }}=\frac{-1 \pm \sqrt{1+\tan (\boldsymbol{\beta}-\mathrm{FOV} / 2)^{2}}}{\frac{\tan (\boldsymbol{\beta}-\mathrm{FOV} / 2)}{2 \cdot f}}
$$

This gives the diameter of the area of M1 not suffering partial vignetting:

$$
\operatorname{Dnp}_{M 1}=2 \cdot y_{\text {limit_partial }}
$$

Analogously the effective diameter of M2 is increased, as well as the size of the cut from the edge of the baffle. Assuming for convenience the paraxial approximation, we can compute the change in these parameters by considering that in the worst case the angle $\boldsymbol{\alpha}$ is increased by FOV/2. So the increased effective diameter of M2 required because of the finite field of view is:

$$
\text { EffectiveDiameter }_{M 2}=2 \cdot d \cdot \tan (\boldsymbol{\alpha}+\text { FOV } / 2)
$$

And to avoid the partial vignetting the cut at the edge of the buffer size needs to be:

$$
\text { Xcut }_{b}=f-\frac{D_{b}}{\tan (\boldsymbol{\alpha}+\mathrm{FOV} / 2)}
$$

The FOV used for the computations is ACAM's field of 8.3 arcmin .

## 5. Results of computations

The computations have been done using an AWK program, this program is listed in appendix A. The program follows the formulas developed in sections 3 and 4 above. The execution gives the following results:

```
M1: D=4.18 m, f= 10.44 m
d(M1-M2)=8.035 m, d(M2-f) = 2.405 m
d_measured(border_M2-baff)=0.669 m
R(M2) = 6.23 m, heigth(M2-border ) = 0.020 m
d_estimated(M2-baff)=0.649 m
*** vignetting of axial rays ***
a1pha= 11.432 deg
Diameter effective of M2= 0.973 m
Needed diameter of the baffle= 1.235 m
Diameter not vignetted of M1 with a baffle of 1.212 m= 4.103 m
Percentage of light loss du to vignetting= 4.00 %
Length of baffle not vignetting from the border of M2= 0.611 m
Cut needed in the baffle: 5.8 cm
*** Partial vignetting in the F.O.V. ***
Diameter effective of M2= 0.979 m
Diameter without vignetting (partial) with FOV 8.3 arcmin = 4.077 m
Percentage of M1 total or partially vignetted= 5.30%
Length of baffle not vignetting from the border of M2= 0.593 m
Cut needed in the baffle: 7.6 cm
```


## 6. Zemax computations

Just from the 1.22 m diameter of the central obscuration, a vignetting of $8.5 \%$ of the 4.18 m telescope aperture is expected. Zemax models dating to Jan 2007 had taken account of the central obscuration caused by the M2 support structure and confirmed that the spider vanes caused an additional $\sim 0.6 \%$ vignetting, as mentioned in the Observer's Guide. These models also
predict an almost constant value of $90.8 \%$ of rays not vignetted out to a field radius of about 9 arcminutes, where the gradual fall-off brings the value below $90.5 \%$.

However this obscuration affects only parallel rays coming from the sky, while the aperture presented by the baffle in front of M2 was not modelled in any of the relevant Zemax models. Models of scattered moonlight at CASS from 2011 did include baffles, but being non-sequential models they did not allow vignetting calculations.

For this purpose, the M2 baffle can be modelled in Zemax as a circular aperture of Maximum Radius 606 mm (from the measured 1212 mm diameter) on a surface at 649 mm from M2 (measured depth of baffle). The vignetting was then calculated with a higher ray density and field density $(300,30)$ than default $(10,20$ respectively). This reduced the overall throughput (above these density values no significant change is observed) but is more accurate. The correct location for the system stop is at the baffle aperture.

The vignetting thus calculated is summarised in the following table, which reproduces the fraction of rays not vignetted, both with and without the baffle in the model, for field radius out to 6 arcmin at Cassegrain. The last column shows the percentage increase in throughput expected for a baffle of optimised dimensions, in agreement with the calculations in the Section 5. The maximum field radius for ACAM is highlighted.

| $\begin{array}{l}\text { Field } \\ \text { radius } \\ \text { (arcmin) }\end{array}$ | $\begin{array}{l}\text { Fraction of rays } \\ \text { not vignetted }\end{array}$ |  | $\begin{array}{l}\text { Difference } \\ \text { hitherto } \\ \text { baffle }\end{array}$ | $\begin{array}{l}\text { Obscuration } \\ \text { only }\end{array}$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| oimprovement |  |  |  |  |
| achievable |  |  |  |  |$]$

From the above table it is clear that the baffle would not be expected to cause any noticeable variation in flat field intensity over the ACAM field. Zemax also confirms the length, calculated
in Section 5, by which the baffle depth should be reduced in order to avoid vignetting on axis. Removing 5.8 cm is just enough to make the beam diameter from on-axis rays, as focused by M1, smaller than the 1212 mm diameter of the baffle. At the edge of the ACAM field, the losses are just $0.6 \%$ more than on-axis with this reduction in baffle length.

## 7. Conclusions

We have shown that the M2 baffle is limiting the effective aperture of the WHT to 4.10 m for on-axis observations (at all focal stations except Prime focus) at to 4.07 m at the edge of the ACAM field radius of 4.2 arcminutes.

We considered the possibility that the M2 baffle was intended to obscure the outer parts of the primary mirror for reasons such as poor figuring or infra-red observations. However nothing in the documentation available would support these arguments. In the original contract document signed by Grubb-Parsons and RGO (Ref. 2), measurements of the diameter of the upper part of the mirror blank is given as 4220 mm diameter with a chamfer of unspecified size. We estimate this chamfer as removing about 10 mm from the mirror front surface. The document goes on to specify that the figured surface of the mirror should extend to within 20 mm of the chamfer. All of this allows us to assert that the figured surface is no smaller than 4180 mm diameter (it may be up to 10 mm wider, depending on the effect of the chamfer).

Indeed all relevant Zemax models of the telescope have recorded it as 4180 mm . This means, for example, that all throughput calculations are based on this telescope aperture, whereas in fact it is $3.5 \%$ smaller in area.

It is true that infra-red telescopes usually place the aperture stop at the secondary mirror to avoid radiation from support structures. However this stop is not the same as the conventional baffle installed around WHT M2, but (e.g. at the ESO 4.5 m NTT, see Ref. 3) a ring of low emissivity material placed immediately over M2, tilted to reflect IR from the primary off to the side. It does not seem the M2 baffle fulfills such a function.

Since there are no elements in the M2 support structure wider than the outer diameter of the baffle, the solution of using a wider baffle to increase throughput is offset by the concomitant increase in the central obscuration.

Hence we recommend that a baffle up to 76 mm shorter than currently installed be manufactured and tested for use, particularly with wide field instruments such as ACAM, including checks that scattered light is not increased.

## 8. References

1. ING Observers' Guide, 1995
2. Specification for figuring and polishing of a 4.2 m telescope mirror - RGO contract document, 1980
3. Reflecting Telescope Optics II, R.N.Wilson, p464

## 9. Appendix A: computation program

```
# Program to compute the limits of vignetting by M2 baffle at WHT
# Diego Cano Infantes 6-XI-12
# Usage: echo 1 | awk -f vignet.awk
# We have measured the diameter of the cylindrical surface of the
# baffle surrounding the secondary mirror.
# The diameter si 121 cm and the distance between M2 and the edge of
# the baffle is 66 cm.
# The diameter of the primary mirror m1 is 4.18 m and its focal 10.44 m
# The distance M1-M2 is |M1-M2|=8.035 m, and so the distance from M2
# to the focus of M1 is: d=f-|M1-M2| = 2.405 m
# The equation describing the shape of the parabolic mirror is y^2=4fx
# The tangent of the angle alpha that forms the ray reflected in the
# mirror M1 border with respect to the horizontal is tan(alpha)=y/(f-x)
# With is angle alpha we can compute what is the effective diameter of
# the secondary mirror (M2) as well as if the protection baffle is
# vignetting or not any area of the primary mirror.
function abs(x) {
    if (x < 0) return -x;
    else return x;
    }
function tan(x) {
    return sin(x)/cos(x);
    }
BEGIN
    # using the data from "Observer's Guide" pg 36, table 2.2
    f=10.4396 # focal distance of M1
    D=4.18 # diameter of M1 4.18 m (no 4.2 !!!)
    Fratio=f/D # F# de M1
    d=f-8.035 # distance M2 to focus of M1 (Zemax mode1)
    # distance between M2 to the baffle edge
    # ( the measures here goes from the baffle edge to the support
    # ring of M2, plus the disntace fromt he support to the
    # border of the M2 mirror )
    dM2baff=(659+13.2 + 657+8.7 + 658+9 + 659+11)*1.E-3/4;
    # diameter of baffle
    diabaff=(1207.5 + 1211 + 1211 + 1215 + 1212 + 1213)*1.E-3/6;
    printf("M1: D=%.2f m, f= %.2f m\n", D, f)
    printf("d(M1-M2)=8.035 m, d(M2-f)=%.3f m\n", d)
    printf("d_measured(border_M2-baff)=%.3f m\n", dM2baff)
    # M2 changes the Fnumber from F2.5 to F11, so assuming it is
    # spheric, we have: 1/d' - 1/d = 1/f2, being |f2|=R2/2
    # we also have: d'/d = 11/2.5
    dp=d*10.94/Fratio
    f2= 1/(1/dp-1/d)
    R2 = 2*abs(f2)
    DM2=1.001 # diameter of M2
    # the heigth of the center with respect to the border is:
    h= R2 - sqrt( R2^2 - (DM2/2)^2)
    printf("R(M2)= %.2f m, heigth(M2-border)= %.3f m\n", R2, h)
    dM2baff -= h
    printf("d_estimated(M2-baff)= %.3f m\n", dM2baff)
    DEG=45/atan2(1,1) # conversion factor'rad--->deg
    FOV=8.3/60/DEG # field of view de ACAM in radians
    }
```

```
\{
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# COMPUTATION FOR AXIAL RAYS \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
\(\mathrm{y}=\mathrm{D} / 2\)
\(\mathrm{x}=\mathrm{y} * \mathrm{y} /(4 * \mathrm{f})\) \# ( \(\mathrm{x}, \mathrm{y})\) is the pint at the border of m1 mirror
talfa \(=y /(f-x) \quad\) \# tangent of limit angle
deM2= d*talfa*2 \# effective diameter of M2
dnbaff \(=(d+d M 2 b a f f) *\) talfa*2 \# needed diameter of the baffle
printf("\\n*** Vignetting of axial rays ***"n")
printf("alpha= \%.3f deg\n", DEG*atan2(talfa,1)
printf("Diameter effective of \(\mathrm{M} 2=\% .3 \mathrm{f} \mathrm{m} \backslash \mathrm{n} "\), dem2)
printf("Needed diameter of the baffle= \%.3f'm\n", dnbaff)
\# other perspective: if we see what is the angle not blocked by the baffle
tbeta=(diabaff \(/ 2\) )/(d+dM2baff)
\# to compute the semi-diamter ( \(y\) ) of M1 not affected by the baffle
\# it is: theta \(=\mathrm{y} /(\mathrm{f}-\mathrm{y} \wedge 2 /(4 * \mathrm{f})\) ); this gives a second order equation
\# for 'y' with the form:
\# y^2*tbeta/4f \(+\mathrm{y}-\mathrm{f}\) "tbeta \(=0\)
\(y=(-1+\) sqrt(1+tbeta*tbeta)) /(tbeta/2/f)
dnvM1=2*y
printf("Diameter not vignetted of m1 with a baffle of \(\% .3 \mathrm{f}=\% .3 \mathrm{f} \mathrm{m}\) n", diabaff,
dnvM1)
printf("Percentage of light loss due to vignetting= \%.2f \%\%\n", 100*(1-(dnvM1^2-
diabaff^2)/(D^2-diabaff^2)))
\# lets compute now how much we need to cut from the baffle edge
\(x=\) diabaff \(/(2 * t a l f a)-d\);
printf("Length not vignetting from the border of \(\mathrm{M} 2=\% .3 \mathrm{f} \mathrm{m} \backslash \mathrm{n} ", x+h\) )
printf("Cut needed in the baffle: \%.1f cm\n", 100*(dM2baff-x))
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# PARTIAL VIGNETTING in the F.O.V. \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
printf("\n\n*** Partial vignetting in the F.o.v. ***
\(\operatorname{talfa}=\tan (\operatorname{atan} 2(t a l f a, 1)+\mathrm{FOV} / 2)\)
deM2= d*talfa*2 \# effective diameter of M2
printf("Diameter effective of \(\mathrm{M} 2=\% .3 \mathrm{f} \mathrm{m} \backslash \mathrm{n} "\), dem2)
tbeta=tan( \(\operatorname{atan} 2(t b e t a, 1)-F O V / 2)\)
\(\mathrm{y}=(-1+\operatorname{sqrt}(1+\) tbeta*tbeta) \() /(\) tbeta \(/ 2 / \mathrm{f})\)
dnvM1=2*y
printf("Diameter without vignetting (partial) with \(\operatorname{Fov} \% .1 \mathrm{f}\) arcmin \(=\% .3 \mathrm{f} \mathrm{m}\) n",
FOV*60*DEG, dnvM1)
printf("Percentage of M1 total or partially vignetted= \%.2f \%\%\n", 100*(1-(dnvM1^2-
diabaff \(\wedge 2) /(\mathrm{D} \wedge 2\)-diabaff \(\wedge 2)\) ))
\(x=\) diabaff \(/\left(2^{*}\right.\) talfa) \(-d\);
printf("Length not vignetting from the border of \(\mathrm{M} 2=\% .3 \mathrm{f} \mathrm{m} \backslash \mathrm{n} ", x+h\) )
printf("Cut needed in the baffle: \%.1f cm \(\backslash \mathrm{n} ", 100 \%(d m 2 b a f f-x))\)
\}
```

